Combining Recursive Least Square and Principal Component Analysis for Assisted History Matching

by

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13697

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CERTIFICATION OF APPROVAL

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CERTIFICATION OF ORIGINALITY

This is to certify that I am responsible for the work submitted in this project, that the original work is my own except as specified in the reference and acknowledgements, and that the original work contained herein have not been undertaken or done by unspecified sources or persons.

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NURUL SYAZA BINTI MD ANUAR

ABSTRACT

History matching is a process of altering parameters in a reservoir simulator in order to match production performance with observed historical data. There are two methods of history matching; manual history matching, which is the most common approach, and another method is assisted history matching. However, most of the reservoirs in reality are heterogeneous and it requires experience and time to rely on trial and error methods. Assisted history matching can be an alternative solution in saving time thus, using optimization process needs to be further developed and improved to be implemented.

The main objective of this project is to investigate the applicability of Recursive Least Squares (RLS) for parameter estimation methods in assisted history matching. Currently not much attention is given for using RLS for history matching purposes. Even though RLS is a simple and effective method to estimate parameters, RLS have stability problem when number of parameters is high. Therefore, in this project, Principal Component Analysis (PCA) is used to reduce the number of parameters.

The project is divided to several steps; in which the first step is to develop a conceptual model which can be used to generate both synthetic historical data and also simulated data. Forward model was also involved in the process of defining the objective function**.** Next, using simulated data together with historical data, objective function will be computed. This project will study the applicability of the combined algorithm for history matching problem.

The study conducted on PCA and RLS method shows high chances of success in applying these methods for history matching problem. The algorithm formulated also can easily be practiced, provided with ample knowledge of numerical computational tool to implement it. When RLS and PCA are applied, the result obtained with estimated parameter results in lower mean squared error (MSE) between historical and matched result which is 0.75% compared to MSE between historical and simulated which is 5.17%. This proves that when RLS is applied, almost 5% of error can be reduced and thus can result in better forecasting of the reservoir production performance.

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INTRODUCTION

1.1 Background of Study

1.1.1 History Matching

Reservoirs are complex with many unknown parameters and uncertainties that it is difficult to obtain highest accurate details of the reservoir description itself. By history matching, an understanding of the actual reservoir can be obtained. History matching is a process of altering parameters of a simulation model to replicate & match production performance with the actual reservoir. This can be done by comparing the measured or observed production or pressure data with the simulated data from the simulation model. The observed data is also called as historical data. The goal for history matching is for forecasting prediction of reservoir performance to be used in investment strategy and also for reservoir management. According to Satter et al. [1], reservoir management is to get maximum profits from a reservoir by optimizing recovery on the basis of information, knowledge and through management practices.

Generally, history matching has always been applied manually. Manual history matching is defined as reservoir engineers or simulation engineers apply manual adjustments of certain parameters to obtain satisfactory match between the historical data with the simulated data. However, reservoirs are usually heterogeneous and it is not practical to rely on trial and error methods due to the long period of completion and usually with many uncertainties.

1.1.2 Assisted history matching

In 1965, Jacquard [2] introduced first method for assisted history matching. Semi-automatic or assisted history matching is defined as development of a forward reservoir model with an initial approximation of reservoir parameters [3]. In recent years, increasing attention has been given to assisted history matching methods. Generally this method relies on formal optimization schemes in which a particular objective function is minimized by altering the relevant parameters. This is also called as objective function in which it represents the mismatch between historical and simulated response.

Basically, according to Cancelliere et al. [4], reservoir engineers can depend on optimization tools to know better the parameter space and to reduce the time taken for the convergence of the solutions. Optimization tools for assisted history matching are given by estimating uncertain parameters from indirect measurements. A few methods of parameter estimation that is available, such as genetic algorithm and ensemble Kalman filter, which latter have proven to be very successful for usage in history matching.

Another technique of assisted history matching is by describing the reservoir using fewer parameters or what we called as re-parameterization. Re-parameterization of the reservoir is used for speeding-up the optimization by reducing the number of parameters. There are few methods which have been used to reduce the number of parameters, which are gradzone analysis, zonation, principal component analysis, and also discrete cosine transform as explained in [5].

1.1.3 Inverse Problem

Main function of history matching is to match direct observations of the reservoir (production data) with the simulated model. This can be achieved by changing parameters and thus, forecast of the reservoir performance can be done for the future. The observations are used to predict or determine the parameters or variables that describe the system. This is called as inverse problem. Inverse problem starts by an answer already known, however, the question is not known. Therefore in this case, given with the observed data of the system, the properties of the system can be determined. An inverse problem of interest to reservoir engineers is history matching. The rock properties or parameters will be estimated using information gained from production data. However, it is difficult to directly compute the relationship between measurements such as water–cut or pressure; with parameters such as permeability or porosity due to its nonlinear relationship.

Another concern that arises with inverse problem is that there is no unique solution. Basically there are an infinite number of solutions that can exist to fit into giving the same answer as the observed response. Example in Oliver et al. [6] illustrates clearly that the inverse problem of determining the gridblock porosities and permeabilities from flowing wellbore pressure will not have a unique solution when the data are inaccurate and measurements are obtained at only a few locations. In fact, there are actually two different set of permeability values that can equally honor the observed pressure data well. Even though the parameters can be reduced, the solutions for the inverse problem are still not unique.

1.1.4 Uncertainty in parameter estimation

Any numerical model of a physical system is basically only an approximation. There will always be uncertainty in an estimate prediction. This is due to some error in measurements or also different numerical schemes and assumptions used to solve the equations of the model. The uncertainty in a prediction can be classified into modeling error and measurement error [7].

1.1.5 Recursive Least Squares (RLS) & Principal Component Analysis (PCA) method

In recent years, more number of papers on history matching was published showing increase in interest for history matching as researched by Rwechungura et al. [5]. Using manual history matching has always been the only solution, but with development of assisted history matching, time taken for history matching process can be saved. In this paper, the optimization process using Recursive Least Squares (RLS) method and parameter reduction method of Principal Component Analysis (PCA) will be used.

1.2 Problem statement

Manual history matching usually is time-consuming and expensive. Large number of parameters or uncertainties can be difficult for the reservoir engineer. Higher skills or more experienced engineer will be needed to handle history matching. Using alternative methods such as automated or assisted history matching, larger datasets of parameters can be handled semi-automatically. Thus, shorter time can be expected to match the simulation model with the actual reservoir.

At the present time, the application of Recursive Least Squares (RLS) technique is not widely used for optimization of assisted history matching. This project will be using RLS as parameter estimation method for assisted history matching. RLS algorithm is known to pursue fast convergence and can perform well in time-varying environments [8]. However, Cioffi [9] pointed out that RLS has some stability problems when the number of parameters involved is high. Therefore a smaller number of parameters would be preferred. PCA method will be combined with RLS in order to reduce the parameters.

Significance of the applicability of RLS method for assisted history matching will be proven, if applied in a real reservoir. However, as it is an undergraduate project with limited time frame and limitations of data, it can only be applied to a synthetic model for now. With simpler algorithm that will not take much time and space, RLS method have the potential able to be developed as one of assisted history matching technique.

1.3 Objectives of the Study

The main objective of this project is to investigate the applicability of Recursive Least Squares (RLS) as an optimization process for assisted history matching purpose. The RLS technique will be combined with a parameter reduction method which is Principal Component Analysis (PCA). An algorithm that combines these both methods will be developed at the end of this project. The algorithm then can be used for input in a numerical computational tool. In order to achieve the main objectives, a few specific objectives have been defined to be achieved before obtaining the main objective.

1. To develop a simple conceptual model that characterizes features of real reservoir with two different datasets of parameters
2. To generate synthetic historical data
3. To generate simulated data
4. To derive forward model and objective function
5. To study the application of PCA and RLS methods for history matching and in other disciplines
6. To develop algorithm combining RLS and PCA method
7. To apply PCA and RLS methods

1.4 Scope of the study

Main scope of this project is to combine the parameter estimation technique, RLS algorithm with the parameter reduction methods, which is PCA to be used for history matching purpose. Reservoir parameters that are to be estimated as a variable for each block will be limited only to permeability. This project applied PCA and RLS in a synthetic model. Any reservoir simulator (ECLIPSE, IMEX) will be used to develop the synthetic model and to simulate performance of the model. Numerical computational tool (MATLAB, FORTRAN) will be used for the application of RLS and PCA in reducing & estimating the parameters.

CHAPTER 2
LITERATURE REVIEW

2.1 History matching

As a fundamental step in order to get forecasting of reservoir production and quantifying the uncertainty, history matching helps in developing a satisfactory and accurate model that produces similar to the reservoir. There are mainly three types of history matching; manual, automatic; and assisted history matching. According to Mattax and Dalton [10], due to the complexity of the reservoir, manual history matching can be time-consuming, expensive and often frustrating although it was commonly applied until now. These problems lead to an alternative or improvement of history matching procedures which is automatic history matching. Automatic history matching generally uses nonlinear optimization methods to achieve a best match or fit of the historical data.

However, a statement in [4] reflects that it is not quite possible to really rely on automatic history matching since it is not quite possible to identify the best optimization methodology that can solve for a wide range of reservoirs. This is due to the non-uniqueness of the reservoir model, in which there can be several combinations of parameters that capable to adequately match the behavior of the system. Some drawbacks of automatic history matching are also pointed out, which there are some limitations for the optimization methods and the high expense of the techniques [10]. Computer costs can results in higher expense compared to the personnel cost.

Assisted history matching meanwhile brings the idea of relying on optimization tools to better explore parameter space and to decrease the time taken for the parameters to converge with the real parameters or solutions. However, the control is still done by a reservoir engineer. For development to improve manual history matching, research on assisted history matching techniques are very important and beneficial to the industry as it can save the time taken to complete history matching process with accurate results.

2.2 Assisted history matching

Semi-automatic or assisted history matching was defined as development of a forward reservoir model with an initial guess of reservoir parameters [3]. By altering the relevant parameters, the model will go through a systematic minimization process of an objective function that represents the mismatch between historical and calculated response.

Forward model is characterized by initial parameters to simulate the behavior of an actual system. Practically, first, the model was estimated and introduced as the key model to be further applied with the optimization algorithms. In addition, for history matching purpose, the forward modeling is also used for defining the objective function.

For objective function, [5] defines it as the quantity of mismatch between the historical data and the simulated data for a given set of parameters. The main objective of assisted history matching is to obtain the smallest differences of historical data and simulated data. Thus, it can be said that the process of history matching is iterative in which the unknown parameters will keep modified to minimize the objective function.

According to [11], the basic procedure of optimization process of history matching is where the initial parameter is first guessed and using the parameter, the simulation will be run. Next, the objective function will be calculated from the misfits of the historical data and the simulated data. If the objective function value is within the satisfied range, then the history matching is considered to be successful. However, if the objective function is not within the satisfied range, next, the parameter will be estimated using optimization methods for the next iteration until converge to the true parameter. Figure 1 below shows the flowchart of the optimization process of history matching.

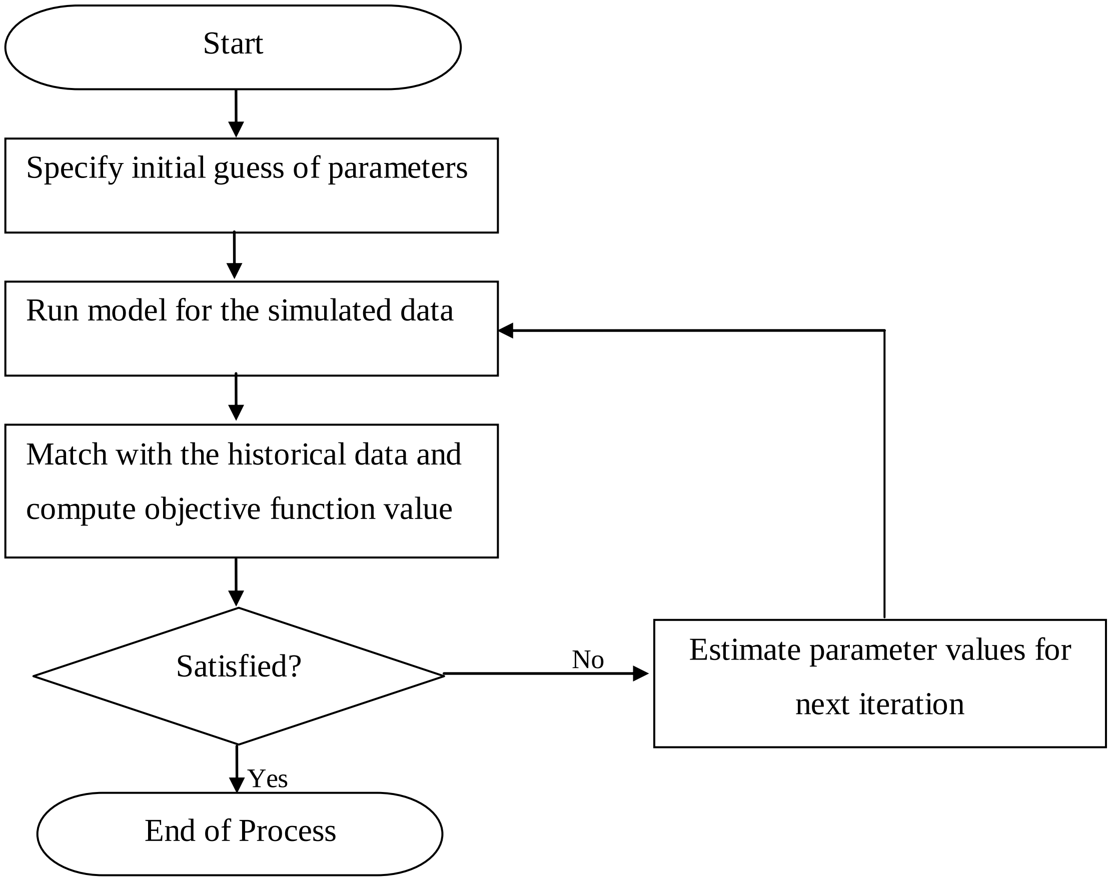

Figure 1 Flowchart of optimization process of history matching (Mata-Lima, 2011)

There are many types of objective functions. A study by Bertolini and Schiozer presented that, different types of objective function influenced the performance of the optimization method [12]. The result from that study is that the least squares formulation, which is among the types of objective function, proved to be the best as it can obtain the best results with a smaller number of simulations.

However, not only least square formulation can be used, there are also other alternative methods in which the levels of details can be adjusted in which it more robust to quantify the variance between images compared to the least square formulation [13]. This application will be useful for more complex and detailed case where they are more difficult in history matching of seismic data using the least squares formulation.

There are actually many optimization methods that can be applied for the history matching purpose in order to estimate the parameters for the next iteration. Some of the methods may come from application in different disciplines or might be discovered particularly in solving for history matching problem. As there are more improvements and applicability of these methods in solving history matching, reservoir engineers can choose the best method for the particular reservoir and easily done the history matching procedure without much time and energy taken and thus can increase the productivity of the personnel.

2.3 Parameter estimation technique: Recursive Least Squares (RLS)

Parameter estimation is defined as an estimation from indirect measurement of uncertain and non-linear parameters such as saturation, pressure, permeability and porosity [5]. Recursive parameter estimates ensure the estimated parameters converge with the 'true' or real parameters of the system with high chance probability. Generally, in order to minimize the objective function, iterative algorithm is used to calculate the unknown parameters by successive approximation. Recursive steps terminate when it converge with true parameters, meanwhile iterative will only stops when the loop-continuation condition fails. Recursive is therefore more likely to be accurate in estimating the parameter. Another characteristic of recursive is that it always considers previous data into calculation. This results in a solution that is dependent with the initial data. Therefore, as the focus of this paper, recursive least squares (RLS) algorithm will be adapted for application of history matching.

Gauss's theory of recursive least squares estimation was rediscovered by Plackett, in 1950 and 1960 in the context of control theory [14]. Many of the RLS approaches described in detail are within the digital signal processing, adaptive control or system identification literature mainly for electrical engineering disciplines. Characteristics of recursive least-squares filter is that it does not involve taking matrix inverses thus, as soon as measurements are taken, estimates are directly available [15]. This means that the recursive estimates are independent of initial conditions.

One of the main concerns in applying RLS method in this project is the ability of implementing the method in a non-linear system. Most of the real systems are nonlinear systems. During year of 2011, Mitsis and Markou claim that the RLS method can be used in non-linear system [16]. In the study performed, RLS was successfully implemented in a non-linear system with consideration of selecting the initial parameter in a reliable manner.

This is further supported by the results in [17], that the author claims that the proposed model based RLS algorithm is sufficient to represent both linear and nonlinear systems. This is due to the assumption of the nonlinear system to be a linear-in-parameters system. From these both findings, it can be concluded, one of RLS method’s drawback was the systems should be used for a small number of points since the method’s working well in a systems with short memory. With lower noise level, parameter accuracies of RLS become higher.

RLS therefore can be applied to estimate parameters for history matching problem as it can be used in non-linear systems, accurate and high in efficiency. These statements serve as a basis for applying RLS method in this project. To maintain the parameter error estimates of RLS, the number of measurement need to be reduced. It was suggested then to integrate the RLS method with any parameter reduction method such as Principal Component Analysis (PCA) or Gradzone method.

2.4 Re-parameterization: Principal Component Analysis (PCA)

For accurate modeling of the reservoir, a detailed reservoir description is needed and usually it requires larger number of parameters. Therefore, it could be as one of the main limitations faced by the reservoir engineer in order to do history matching.

Using PCA, it is possible to reduce the number of parameters. This method can also be called as re-parameterization in which the numbers of parameters have been optimized to be only representing the number of dominating geological patterns. According to Yadav [18], it is possible to obtain a set of acceptable permeability realizations, in which the characteristic patterns inside this realizations could be obtained by PCA.

PCA has been used for history matching purpose as shown in [5], in which PCA was compared with gradzone analysis. Gradzone analysis is also one of parameterization to be used as a speed up attempt for reducing the number of parameters. From the study, it can be concluded that PCA are applicable for re-parameterization in which PCA managed to reduce from 739 parameters to only 100 parameters. For that particular study, author states the PCA only retain the patterns associated to the highest eigenvalues.

Some of the applications of assisted history matching used PCA to transform zero mean and unit variance attribute vectors to create a new set of primary variables from linear combinations of the original primary variables [19]. Meanwhile in study by Scheevel and Payrazyan [20], the PCA approach is used to derive all meaningful seismic attributes in a single coordinated information in order for limiting the unnecessary attribute.

There also have been a few improvements in applying the PCA methods for history matching. Among the studies, all show promising future for PCA methods to be applied in history matching. In both studies made by Sarma et al. [21, 22], a new parameterization techniques was used which is modification of principal component analysis, known as kernel principal component analysis (Kernel PCA). It enables high order statistics of random fields to be preserved; therefore it is more capable to reproduce complex geology. Khaninezhad and Jafarpour [23] combined geologic functions and nongeologic functions, which is PCA and discrete cosine transform (DCT) respectively, as hybrid parameterization, in order to reduce the number of parameters in the absence of prior information or with uncertain prior knowledge. The author also concluded that using hybrid parameterization that combined geologic and nongeologic expansion function is more appropriate than using either of the two alone. However for this project, the geology factor was not much considered therefore, a basic application of PCA method should be enough in order for it to show positive results.

CHAPTER 3
ASSISTED HISTORY MATCHING

3.1 Forward Modeling

Forward model or mathematical model is practically used for the estimation of the unknown parameters in an inverse theory of history matching. This section will cover the basic definitions and also describe the important theoretical background of multiphase flow in porous media.

In this study, the forward model used for the estimation for the unknown parameters will be represented by the reservoir simulator (i.e. ECLIPSE, IMEX) to model the flow of fluid through porous media. It is required to derive the forward model to be able to relate the parameter input with the output. Since permeability is the input for this study, the expected output that can be directly compared for the well performance is the flow rate. This simulator will be used in order to forecast the behavior of the system under conditions stated in earlier section.

Forward model starts with mass balance equation and also by deriving continuity equation using fundamental laws in petroleum engineering such as Darcy's law and many more. Discretization of the parameters would be applied and also considers boundary conditions. Usually deriving three-phase flow would be very complex. Below shows several steps in deriving the forward model based on guidance from Kleppe, J. [24]. Simple derivation of two-phase flows with permeability as variable is shown in Appendix C.

I. Basic Definitions

- Porosity – void space that represents storage capacity of reservoir rocks. It is defined as

$$\phi = \frac{Pore\ volume}{Bulk\ volume} \tag{1}$$

- Permeability – ability to flow or transmit fluids through a rock. The common unit used in petroleum engineering industry is millidarcy (mD).
- Saturation – fractions of the pore space occupied by gas, oil and water.

$$S_p = \frac{Volume\ of\ p}{Pore\ volume} \qquad \text{p = o,w,g} \tag{2}$$

Fluid saturations are expressed as a fraction of the pore volume therefore their summation should always equal to 1

$$S_g + S_o + S_w = 1 \tag{3}$$

II. Fluid System

The term single phase applies to any system with only one phase present in the reservoir. In some cases it may also apply where two phases are present in the reservoir, if one of the phases is immobile, and no mass exchange takes place between the fluids. This is normally the case where immobile water is present with oil or with gas in the reservoir. By regarding the immobile water as a fixed part of the pores, it can be accounted for by reducing porosity and modifying rock compressibility correspondingly.

General form

Thus, for all three fluid systems, the one phase density may be expressed as:

$$r = \frac{constant}{B}, \tag{4}$$

Which is the model used for the demonstration of fluid description in the following single phase flow equations.

III. Continuity Equation

Starts with conservation of mass, it can be formulated across a control element with one fluid of density ρ is flowing through it at velocity *u*:

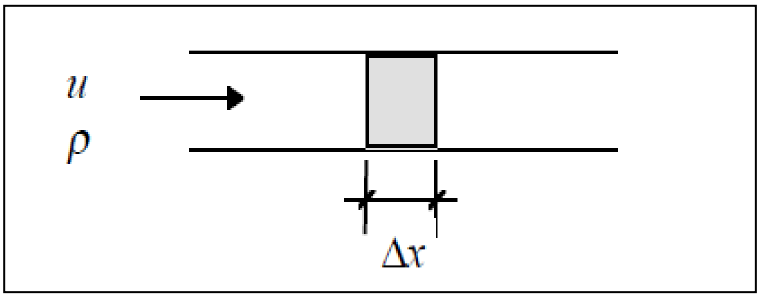

Figure 2 Cross section for conservation of mass

The mass balance for the control element is then written as:

$$\{Mass\ into\ the\ element\ at\ x\} - \{Mass\ out\ of\ the\ element\ at\ x + Dx\} = \{Rate\ of\ change\ of\ mass\ inside\ the\ element\} \tag{5}$$

Or

$$\{u\rho A\}_x - \{u\rho A\}_{x+\Delta x} = \frac{\partial}{\partial t}\{\emptyset A \Delta x \rho\} \tag{6}$$

Dividing by Δx, and taking the limit as Δx goes to zero, continuity equation can be achieved:

$$-\frac{\partial}{\partial x}(A\rho u) = A\frac{\partial}{\partial t}(\emptyset\rho) \tag{7}$$

For constant cross sectional area, the continuity equation simplifies to:

$$-\frac{\partial}{\partial x}(\rho u) = \frac{\partial}{\partial t}(\emptyset\rho) \tag{8}$$

IV. Darcy's Law

Darcy's equation for one dimensional and horizontal flow is:

$$u = -\frac{k}{\mu}\frac{\partial P}{\partial x} \tag{9}$$

V. Differential Equation

Using the fluid model defined above:

$$\rho = \frac{\text{constant}}{B}, \tag{10}$$

By substituting the Darcy's equation and the fluid equation into the continuity equation, the partial differential equation that describes single phase flow in one dimensional porous medium can be obtained:

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right) - q' = \frac{\partial}{\partial t}\left(\frac{\phi}{B}\right) \tag{11}$$

The left hand side of the equation describes fluid flow in the reservoir, and injection/production, while the right hand side represents storage (compressibility's of rock and fluid). In order to bring the right hand side of the equation on a form with pressure as a primary variable, the term will be rearranged before proceeding to the numerical solution.

Chain rule differentiation yields:

$$\frac{\partial}{\partial t}\left(\frac{\phi}{B}\right) = \frac{1}{B}\frac{\partial \phi}{\partial t} + \phi\frac{\partial(1/B)}{\partial t} \tag{12}$$

We will now make use of the compressibility definition for porosity's dependency of pressure at constant temperature:

$$c_r = \frac{1}{\phi}\frac{d\phi}{dP}, \tag{13}$$

or

$$\frac{d\phi}{dP} = \phi c_r, \tag{14}$$

and the fluid model above:

$$\rho = \frac{\text{constant}}{B},$$

which implies that:

$$B = f(P). \tag{15}$$

The right hand side may then be written:

$$\frac{\partial}{\partial t}\left(\frac{\phi}{B}\right) = \frac{1}{B}\frac{\partial \phi}{\partial t} + \phi\frac{\partial(1/B)}{\partial t} = \frac{1}{B}\frac{d\phi}{dP}\frac{\partial P}{\partial t} + \phi\frac{d(1/B)}{dP}\frac{\partial P}{\partial t} = \frac{\phi c_r}{B}\frac{\partial P}{\partial t} + \phi\frac{d(1/B)}{dP}\frac{\partial P}{\partial t} \tag{16}$$

Thus, the flow equation becomes:

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right) - q' = \phi\left[\frac{c_r}{B} + \frac{d(1/B)}{dP}\right]\frac{\partial P}{\partial t} \tag{17}$$

Fluid compressibility may be defined in terms of the formation volume factor as:

$$c_f = B\frac{d(1/B)}{dP} \tag{18}$$

An alternative form of the differential equation is:

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right) - q' = \frac{\phi}{B}\left[c_r + c_f\right]\frac{\partial P}{\partial t} = \frac{\phi c_T}{B}\frac{\partial P}{\partial t} \tag{19}$$

However, normally it is more convenient to use the first form, since fluid compressibility not necessarily is constant, and since formation volume factor vs. pressure data is standard input to reservoir simulators.

VI. Pressure Formulation

We will now use the discretization formulas derived previously to transform our partial differential equation to difference form. For convenience, the time index for unknown pressures will be dropped, so that if no time index is specified, $t+\Delta t$ is implied.

Left side term

The single phase flow term,

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right) \tag{20}$$

is of the form:

$$\frac{\partial}{\partial x}\left[f(x)\frac{\partial P}{\partial x}\right], \tag{21}$$

This derived to be the following approximation:

$$\frac{\partial}{\partial x}\left[f(x)\frac{\partial P}{\partial x}\right]_i = \frac{2f(x)_{i+1/2}\dfrac{(P_{i+1}-P_i)}{(\Delta x_{i+1}+\Delta x_i)} - 2f(x)_{i-1/2}\dfrac{(P_i-P_{i-1})}{(\Delta x_i+\Delta x_{i-1})}}{\Delta x_i} + O(\Delta x). \tag{22}$$

Thus, in terms of the actual flow equation above,

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right)_i = \frac{2\left(\dfrac{k}{\mu B}\right)_{i+1/2}\dfrac{(P_{i+1}-P_i)}{(\Delta x_{i+1}+\Delta x_i)} - 2\left(\dfrac{k}{\mu B}\right)_{i-1/2}\dfrac{(P_i-P_{i-1})}{(\Delta x_i+\Delta x_{i-1})}}{\Delta x_i} + O(\Delta x) \tag{23}$$

We shall now define *transmissibility* as being the coefficient in front of the pressure difference appearing in the approximation above:

<u>Transmissibility in plus direction</u>

$$Tx_{i+1/2} = \frac{2}{\Delta x_i(\Delta x_{i+1}+\Delta x_i)}\left(\frac{k}{\mu B}\right)_{i+1/2} \tag{24}$$

<u>Transmissibility in minus direction</u>

$$Tx_{i-1/2} = \frac{2}{\Delta x_i(\Delta x_{i-1}+\Delta x_i)}\left(\frac{k}{\mu B}\right)_{i-1/2} \tag{25}$$

Then, the difference form of the flow term in the partial differential equation becomes:

$$\frac{\partial}{\partial x}\left(\frac{k}{\mu B}\frac{\partial P}{\partial x}\right)_i \approx Tx_{i+1/2}(P_{i+1}-P_i) + Tx_{i-1/2}(P_{i-1}-P_i) \tag{26}$$

Using $Tx_{i+1/2}$ as example, the transmissibility consists of three groups of parameters:

$$\frac{2}{\Delta x_i(\Delta x_{i+1}+\Delta x_i)} = constant, \tag{27}$$

$$k_{i+1/2} = \bar{k} = f(x), \tag{28}$$

$$\left(\frac{1}{\mu B}\right)_{i+1/2} = \overline{\left(\frac{1}{\mu B}\right)} = f(P). \tag{29}$$

The forms are to be determined of the two latter groups before proceeding to numerical solution. Starting with Darcy's equation:

$$q = -\frac{kA}{\mu B}\frac{\partial P}{\partial x}. \tag{30}$$

For flow between two grid blocks:

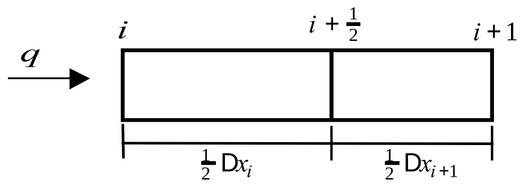

Figure 3 Grid blocks

Assume that the flow is steady state, i.e. *q=constant* and that k is dependent on position. The equation may be rewritten as:

$$q\frac{dx}{k} = -A\frac{dP}{\mu B} \tag{31}$$

<u>Permeability</u>

Integrate the equation above between block centers:

$$q\int_i^{i+1}\frac{dx}{k} = -A\int_i^{i+1}\frac{dP}{\mu B} \tag{32}$$

The left side may be integrated in parts over the two blocks in the discrete system, each having constant permeability:

$$q\int_i^{i+1}\frac{dx}{k} = \frac{q}{2}\left(\frac{\Delta x_i}{k_i}+\frac{\Delta x_{i+1}}{k_{i+1}}\right) \tag{33}$$

Defining an average permeability, $\bar{k}$:

$$\frac{q}{2}\left(\frac{\Delta x_i}{k_i}+\frac{\Delta x_{i+1}}{k_{i+1}}\right) = \frac{q}{2}\frac{\Delta x_i+\Delta x_{i+1}}{\bar{k}} \tag{34}$$

Yielding,

$$\bar{k} = \frac{\Delta x_i + \Delta x_{i+1}}{\left(\frac{\Delta x_i}{k_i} + \frac{\Delta x_{i+1}}{k_{i+1}}\right)} \tag{35}$$

which is the *harmonic average* of the two permeabilities. In terms of the grid block system, following are the expressions for the harmonic averages:

$$\bar{k} = k_{i+1/2} = \frac{\Delta x_{i+1} + \Delta x_i}{\frac{\Delta x_{i+1}}{k_{i+1}} + \frac{\Delta x_i}{k_i}} \tag{36}$$

and

$$\bar{k} = k_{i-1/2} = \frac{\Delta x_{i-1} + \Delta x_i}{\frac{\Delta x_{i-1}}{k_{i-1}} + \frac{\Delta x_i}{k_i}} \tag{37}$$

Fluid mobility term

Integrate the right hand side:

$$-A\int_i^{i+1} \frac{dP}{\mu B} \tag{38}$$

Replacing the fluid parameters by mobility $\lambda = \frac{1}{\mu B}$, and letting λ be a weak function of pressure, and assuming the pressure gradient between the block centers to be constant, the weighted average of the blocks' mobility terms is representative of the average. The average mobility terms are:

$$\lambda_{i+1/2} = \frac{(\Delta x_{i+1}\lambda_{i+1} + \Delta x_i \lambda_i)}{(\Delta x_{i+1} + \Delta x_i)} \tag{39}$$

and

$$\lambda_{i-1/2} = \frac{(\Delta x_{i-1}\lambda_{i-1} + \Delta x_i \lambda_i)}{(\Delta x_{i-1} + \Delta x_i)} \tag{40}$$

Right side term

The discretization of the right side term

$$\phi\left[\frac{c_r}{B} + \frac{d(1/B)}{dP}\right]\frac{\partial P}{\partial t} \tag{41}$$

is done by using the backward difference approximation derived previously:

$$\left(\frac{\partial P}{\partial t}\right)_i \approx \frac{P_i - P_i^t}{\Delta t} \tag{42}$$

Define a storage coefficient as:

$$C_{P_i} = \frac{\phi_i}{\Delta t}\left[\frac{c_r}{B} + \frac{d(1/B)}{dP}\right]_i \tag{43}$$

and the right side approximation becomes:

$$\phi\left[\frac{c_r}{B} + \frac{d(1/B)}{dP}\right]\frac{\partial P}{\partial t} \approx C_{P_i}(P_i - P_i^t) \tag{44}$$

Thus, the difference form of the single phase flow equation is (for convenience, the approximation sign is hereafter replaced by an equal sign):

$$Tx_{i+1/2}(P_{i+1} - P_i) + Tx_{i-1/2}(P_{i-1} - P_i) - q_i' = C_{P_i}(P_i - P_i^t), i = 1, N\ . \tag{45}$$

VII. Boundary and Initial Conditions

In reservoir simulation, the boundary conditions are no flow boundaries at the end faces of the reservoir, and production/injection wells where either rate or pressure are specified, located in any of the grid blocks.

No flow boundaries

No flows at the boundaries are assigned by giving the respective transmissibility a zero value at that point. This is the default condition. For one-dimensional system, this type of condition would for example be applied to the two end blocks so that:

$$Tx_{1/2} = 0$$

$$Tx_{N+1/2} = 0\ .$$

Production/injection wells

We will now introduce a well term in our difference equation, so that it becomes:

$$Tx_{i+1/2}(P_{i+1} - P_i) + Tx_{i-1/2}(P_{i-1} - P_i) - q_i' = C_{P_i}(P_i - P_i^t), i = 1, N \tag{45}$$

This equation applies in one-phase, one-dimensional, horizontal system.

VIII. Three-phase flow

Adding water & gas to the previous for a one-dimensional, horizontal system, we have the following three continuity equations:

$$-\frac{\partial}{\partial x}(\rho_{oL} u_o) = \frac{\partial}{\partial t}(\phi \rho_{oL} S_o)$$

$$-\frac{\partial}{\partial x}(\rho_g u_g + \rho_{oG} u_o) = \frac{\partial}{\partial t}\left[\phi(\rho_g S_g + \rho_{oG} S_o)\right]$$

$$-\frac{\partial}{\partial x}(\rho_w u_w) = \frac{\partial}{\partial t}(\phi \rho_w S_w) \tag{46}$$

and the corresponding Darcy equations for a horizontal system:

$$u_o = -\frac{k k_{ro}}{\mu_o}\frac{\partial P_o}{\partial x}$$

$$u_g = -\frac{k k_{rg}}{\mu_g}\frac{\partial P_g}{\partial x}.$$

$$u_w = -\frac{k k_{rw}}{\mu_w}\frac{\partial P_w}{\partial x}, \tag{47}$$

where

$$P_{cog} = P_g - P_o \tag{48}$$

$$P_{cow} = P_o - P_w \tag{49}$$

$$S_o + S_g + S_w = 1 \tag{50}$$

Black Oil PVT Properties are as defined:

$$\rho_o = \frac{\rho_{oS} + \rho_{gS} R_{so}}{B_o} = \frac{\rho_{oS}}{B_o} + \frac{\rho_{gS} R_{so}}{B_o} = \rho_{oL} + \rho_{oG} \tag{51}$$

$$\rho_g = \frac{\rho_{gS}}{B_g} \tag{52}$$

$$\rho_w = \frac{\rho_{wS}}{B_w} \tag{53}$$

Undersaturated systems

We define an undersaturated system, as before, by:

$$P_o > P_{bp} \tag{54}$$

and

$$S_o = 0. \tag{55}$$

which implies that

$$B_o = f\left(P_o, P_{bp}\right)$$

and

$$R_{so} = f\left(P_{bp}\right).$$

The flow equations become:

$$\frac{\partial}{\partial x}\left(\frac{k k_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right) - q_o' = \frac{\partial}{\partial t}\left(\frac{\phi}{B_o}\right) \tag{56}$$

and

$$\frac{\partial}{\partial x}\left(R_{so}\frac{k k_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right) - q_g' - R_{so} q_o' = \frac{\partial}{\partial t}\left(R_{so}\frac{\phi S_o}{B_o}\right), \tag{57}$$

and

$$\frac{\partial}{\partial x}\left(\frac{kk_{rw}}{\mu B_w}\frac{\partial P_w}{\partial x}\right)-q'_w=\frac{\partial}{\partial t}\left(\frac{\phi S_w}{B_w}\right) \tag{58}$$

Boundary conditions

The boundary conditions for undersaturated oil-gas-water systems are similar to the boundary conditions for undersaturated oil-gas systems. In addition to injection of gas, we may also inject water. Production wells need to account for production of water in addition to oil and solution gas. The appropriate well equations for water and oil production are identical to the ones presented in the oil-water section.

Discrete equations

Developing the discrete equations along the same principles and using similar assumptions as in the previous cases, using P_o, P_{bp} and S_w as the primary variables, we get:

$$T_{xo_{i+1/2}}\left(P_{o_{i+1}}-P_{o_i}\right)+T_{xo_{i-1/2}}\left(P_{o_{i-1}}-P_{o_i}\right)-q'_{oi}$$
$$=C_{poo_i}\left(P_{o_i}-P_{o_i}^t\right)+C_{bpo_i}\left(P_{bp_i}-P_{bp_i}^t\right)+C_{swo_i}\left(S_{w_i}-S_{w_i}^t\right), i=1,N$$
$$\left(R_{so}T_{xo}\right)_{i+1/2}\left(P_{o_{i+1}}-P_{o_i}\right)+\left(R_{so}T_{xo}\right)_{i-1/2}\left(P_{o_{i-1}}-P_{o_i}\right)-\left(R_{so}q'_o\right)_i-q'_{gi}$$
$$=C_{pog_i}\left(P_{o_i}-P_{o_i}^t\right)+C_{bpg_i}\left(P_{bp_i}-P_{bp_i}^t\right)+C_{swg_i}\left(S_{w_i}-S_{w_i}^t\right), i=1,N$$
$$T_{xw_{i+1/2}}\left[\left(P_{o_{i+1}}-P_{o_i}\right)-\left(P_{cow_{i+1}}-P_{cow_i}\right)\right]+T_{xw_{i-1/2}}\left[\left(P_{o_{i-1}}-P_{o_i}\right)-\left(P_{cow_{i-1}}-P_{cow_i}\right)\right]-q'_{wi}$$
$$=C_{pow_i}\left(P_{o_i}-P_{o_i}^t\right)+C_{bpw_i}\left(P_{bp_i}-P_{bp_i}^t\right)+C_{sww_i}\left(S_{w_i}-S_{w_i}^t\right), i=1,N$$

(59)

where

$$T_{xo_{i+1/2}}=\frac{2\lambda_{o_{i+1/2}}}{\Delta x_i\left(\dfrac{\Delta x_{i+1}}{k_{i+1}}+\dfrac{\Delta x_i}{k_i}\right)} \tag{60}$$

$$\lambda_o=\frac{k_{ro}}{\mu_o B_o}$$

$$\lambda_{o_{i+1/2}}=\begin{cases}\lambda_{o_{i+1}} if\ P_{o_{i+1}}\geq P_{o_i}\\ \lambda_{o_i} if\ P_{o_{i+1}}<P_{o_i}\end{cases}$$

$$R_{so_{i+1/2}}=\begin{cases}R_{so_{i+1}} if\ P_{o_{i+1}}\geq P_{o_i}\\ R_{so_i} if\ P_{o_{i+1}}<P_{o_i}\end{cases}$$

and

$$C_{poo_i} = \frac{\phi_i(1-S_{w_i})}{\Delta t}\left(\frac{c_r}{B_o} + \frac{\partial(1/B_o)}{\partial P_o}\right)_i \tag{61}$$

$$C_{bpo_i} = \frac{\phi_i(1-S_{w_i})}{\Delta t}\left(\frac{\partial(1/B_o)}{\partial P_{bp}}\right)_i \tag{62}$$

$$C_{swo_i} = -\frac{\phi_i}{B_{oi}\Delta t} \tag{63}$$

$$C_{pog_i} = \frac{(R_{so}\phi)_i(1-S_{w_i})}{\Delta t}\left(\frac{c_r}{B_o} + \frac{\partial(1/B_o)}{\partial P_o}\right)_i \tag{64}$$

$$C_{bpg_i} = \frac{\phi_i(1-S_{w_i})}{\Delta t}\left[R_{so}\frac{\partial(1/B_o)}{\partial P_{bp}} + \frac{1}{B_o}\frac{dR_{so}}{dP_{bp}}\right]_i \tag{65}$$

$$C_{swg_i} = -\frac{\phi_i R_{so_i}}{B_{oi}\Delta t} \tag{66}$$

$$C_{pow_i} = \frac{\phi_i S_{w_i}}{\Delta t}\left(\frac{c_r}{B_w} + \frac{\partial(1/B_w)}{\partial P_w}\right)_i \tag{67}$$

$$C_{bpw_i} = 0 \tag{68}$$

$$C_{sww_i} = \frac{\phi_i}{B_{wi}\Delta t} - \frac{\phi_i S_{w_i}}{\Delta t}\left(\frac{c_r}{B_w} + \frac{d(1/B_w)}{dP_w}\right)_i\left(\frac{dP_{cow}}{dS_w}\right)_i \tag{69}$$

The derivative terms to be computed numerically for each time step based on the input table to the model, now are:

$$\left(\frac{\partial(1/B_o)}{\partial P_o}\right)_i, \left(\frac{\partial(1/B_o)}{\partial P_{bp}}\right)_i, \left(\frac{d(1/B_w)}{dP_w}\right)_i, \left(\frac{dR_{so}}{dP_{bp}}\right)_i \, and \left(\frac{dP_{cow}}{dS_w}\right)_i \tag{70}$$

IMPES solution

For an IMPES solution of this system of equations, assumptions equivalent to the ones made in the previous cases are made, namely

$T_{xo}{}^t, T_{xw}{}^t$

$R_{so}{}^t, P_{cow_i}^t$

$C_{poo}{}^t, C_{pog}{}^t, C_{pow}{}^t$

$C_{bpo}{}^t, C_{bpg}{}^t, C_{bpg}{}^t$

$C_{swo}{}^t, C_{swg}{}^t, C_{sww}{}^t$

resulting in the following pressure equation

$$
\begin{aligned}
&\left[Txo^{t}_{i+1/2} + \alpha_i \left(R_{so}T_{xo}\right)^{t}_{i+1/2} + \beta_i Txw^{t}_{i+1/2}\right]\left(Po_{i+1} - Po_{i}\right) + \\
&\left[Txo^{t}_{i-1/2} + \alpha_i \left(R_{so}T_{xo}\right)^{t}_{i-1/2} + \beta_i Txw^{t}_{i-1/2}\right]\left(Po_{i-1} - Po_{i}\right) \\
&- \beta_i Txw^{t}_{i+1/2}\left(Pcow_{i+1} - Pcow_{i}\right)^{t} - \beta_i Txw^{t}_{i-1/2}\left(Pcow_{i-1} - Pcow_{i}\right)^{t} \\
&- q'_{oi} - \alpha_i \left(q'_g + R_{so}{}^{t} q'_o\right)_i - \beta_i q'_{wi} = \\
&\left(Cpoo^{t}_{i} + \alpha_i Cpog^{t}_{i} + \beta_i Cpow^{t}_{i}\right)\left(Po_{i} - Po^{t}_{i}\right), i = 1, N
\end{aligned}
\tag{71}
$$

where

$$
\alpha_i = -Cbpo^{t}_{i} / Cbpg^{t}_{i} \tag{72}
$$

$$
\beta_i = \frac{Cswo^{t}_{i}}{Csww^{t}_{i}}\left(\frac{Cswg^{t}_{i}Cbpo^{t}_{i}}{Cswo^{t}_{i}Cbpg^{t}_{i}} - 1\right) \tag{73}
$$

Rewriting the pressure equation on the familiar form

$$
a_i Po_{i-1} + b_i Po_{i} + c_i Po_{i+1} = d_i, \quad i = 1, N \tag{74}
$$

we may solve for oil pressure by, for instance, as before, Gaussian elimination. Then, having obtained the oil pressures, we may combine the equations above to solve for bubble point pressures and water saturations. If the water equation is used for water saturation, since bubble point pressure does not enter this equation, and the oil equation for the bubble point pressures, we get the following explicit expressions:

$$
\begin{aligned}
Sw_{i} = Sw^{t}_{i} + \frac{1}{Csww^{t}_{i}}\Big[\; &Txw^{t}_{i+1/2}\left[\left(Po_{i+1} - Po_{i}\right) - \left(Pcow_{i+1} - Pcow_{i}\right)^{t}\right] + Txw^{t}_{i-1/2}\left[\left(Po_{i-1} - Po_{i}\right) - \left(Pcow_{i-1} - Pcow_{i}\right)^{t}\right] \\
&- q'_{wi} - Cpow^{t}_{i}\left(Po_{i} - Po^{t}_{i}\right) \;\Big], i = 1, N
\end{aligned}
\tag{75}
$$

$$
\begin{aligned}
Pbp_{i} = Pbp^{t}_{i} + \frac{1}{Cbpo^{t}_{i}}\Big[\; &Txo^{t}_{i+1/2}\left(Po_{i+1} - Po_{i}\right) + Txo^{t}_{i-1/2}\left(Po_{i-1} - Po_{i}\right) - q'_{oi} \\
&- Cpoo^{t}_{i}\left(Po_{i} - Po^{t}_{i}\right) - Cswo_{i}\left(Sw_{i} - Sw^{t}_{i}\right) \;\Big], i = 1, N
\end{aligned}
\tag{76}
$$

3.2 Objective function

Objective function is defined as the quantity of mismatch between the historical data with the simulated data for a given set of parameters [5].

The most common formulation of objective function is based on least squares method as stated in [13] and [12]. Below shows the definition of the objective function in different ways depends on the nature of the problem.

- Least-Square Formulation:

$$F = (d^{obs} - d^{cal})^T (d^{obs} - d^{cal}) \tag{77}$$

- Weighted Least-Square Formulation:

$$F = (d^{obs} - d^{cal})^T w (d^{obs} - d^{cal}) \tag{78}$$

- Generalized Least-Square Formulation:

$$F = \frac{1}{2}(1-\beta)\{(d^{obs} - d^{cal})^T C_d^{-1} (d^{obs} - d^{cal})\} + \frac{1}{2}\beta\left\{\left(\alpha - \alpha_{prior}\right)^T C_\alpha^{-1} (\alpha - \alpha_{prior})\right\} \tag{79}$$

where d^{obs} represents the observed data and d^{cal} as the calculated data as predicted by the forward modeling and w is a diagonal matrix that set the individual weights to each measurement. The weighting factor,$\boldsymbol{\beta}$**,** expresses the relative strength in the initial model and C_d is the covariance matrix of the data and gives information about the correlation among the data. C_α is the covariance matrix of the parameters of the model.

For weighted least-square formulation, the method is based on Gauss-Newton. The formulation has proven to be capable of improving the linear and nonlinear function correlation with a reasonable number of simulations. This project will be using formulation of the simple least-squares. A study by Chen et al. [25] has demonstrated a significant saving in computing time. Based on their observations, the simple least-squares objective function obtained the best performance in the history matching process. In this project, the objective function can be calculated as shown as follows, $O(m) = (d_{obs} - d_{sim})^T (d_{obs} - d_{sim})$ (80)

Where d_{obs} represents observed data or historical data, and d_{sim} represents the simulated data as predicted by the forward modeling.

CHAPTER 4
RECURSIVE LEAST SQUARES ALGORITHM

RLS algorithm

Recursive identification is picked when it is preferable to perform the identification on the spot, such as in adaptive control. Examples of identification methods as implemented in a recursive fashion, such as the parameter estimate at time *t* should be computed as a function of the estimate at time *t*-1 and of the incoming information at time *t*.

Based on a study by Murad [26]on RLS application the model equation is using finite differences for the partial derivatives, for a uniform time and space grid

$$P_j^{(n+1)} = P_j^n + \alpha_j \frac{\Delta t}{\Delta x^2}\left\{P_{j+1}^{(n)} - 2P_j^{(n)} + P_{j-1}^{(n)}\right\} + \frac{\Delta t}{4\Delta x^2}\{\alpha_{j+1} - \alpha_{j-1}\}\left\{P_{j+1}^{(n)} - P_{j-1}^{(n)}\right\} \tag{81}$$

. Substituting in the least square objective function

$$J = \sum_{n=1}^{t}\{P_j^{obs,n} - P_j^{calc,n}\}^2 \tag{82}$$

$$\sum_{n=1}^{t}[P_j^{obs,n} - P_j^{(n-1)}]\left[P_{j+1}^{(n-1)} - 2P_j^{(n-1)} + P_{j-1}^{(n-1)}\right] - \alpha_j \frac{\Delta t}{\Delta x^2}\sum_{n=1}^{t}\left[P_{j+1}^{(n-1)} - 2P_j^{(n-1)} + P_{j-1}^{(n-1)}\right]^2 - \alpha_{j+1}\frac{\Delta t}{4\Delta x^2}\sum_{n=1}^{t}[P_{j+1}^{(n-1)} - P_{j-1}^{(n-1)}]\left[P_{j+1}^{(n-1)} - 2P_j^{(n-1)} + P_{j-1}^{(n-1)}\right] + \alpha_{j-1}\frac{\Delta t}{4\Delta x^2}\sum_{n=1}^{t}[P_{j+1}^{(n-1)} - P_{j-1}^{(n-1)}]\left[P_{j+1}^{(n-1)} - 2P_j^{(n-1)} + P_{j-1}^{(n-1)}\right] = 0 \tag{83}$$

Equation can be written as

$$R_j(t) = -\alpha_{j-1}(t)F_{j,j-1}(t) + \alpha_j(t)F_{j.j}(t) + \alpha_{j+1}(t)F_{j,j+1}(t) \tag{84}$$

To develop a recursive algorithm, the intermediate result is

$$R_j(t+1) = R_j(t) + \left[P_{j+1}^{(t)} - 2P_j^{(t)} + P_{j-1}^{(t)}\right]\left(P_j^{obs,(t+1)} - P_j^{(t)}\right) - \alpha_j \frac{\Delta t}{\Delta x^2}\left[P_{j+1}^{(t)} - 2P_j^{(t)} + P_{j-1}^{(t)}\right] - \frac{\Delta t}{4\Delta x^2}\left(\alpha_{j+1} - \alpha_{j-1}\right)\left[P_{j+1}^{(t)} - P_{j-1}^{(t)}\right] + \alpha_j \frac{\Delta t}{\Delta x^2}\left[P_{j+1}^{(t)} - 2P_j^{(t)} + P_{j-1}^{(t)}\right]^2 - \alpha_{j-1}\frac{\Delta t}{4\Delta x^2}\left[P_{j+1}^{(t)} - P_{j-1}^{(t)}\right]\left[P_{j+1}^{(t)} - 2P_j^{(t)} + P_{j-1}^{(t)}\right] + \alpha_{j+1}\frac{\Delta t}{4\Delta x^2}[P_{j+1}^{(t)} - P_{j-1}^{(t)}]\left[P_{j+1}^{(t)} - 2P_j^{(t)} + P_{j-1}^{(t)}\right] \tag{85}$$

Defining the error,

$$\varepsilon_j(t+1) \equiv \left(P_j^{obs,(t+1)} - P_j^{(t)}\right) - \alpha_j \frac{\Delta t}{\Delta x^2}\left[P_{j+1}^{(t)} - 2P_j^{(t)} + P_{j-1}^{(t)}\right] - \frac{\Delta t}{4\Delta x^2}(\alpha_{j+1} - \alpha_{j-1})\left[P_{j+1}^{(t)} - P_{j-1}^{(t)}\right] \quad (86)$$

The following system of equations is obtained

$$\underline{R}(t+1) = \underline{R}(t) + \underline{D}(t+1)\underline{\varepsilon}(t+1) + \underline{T}(t+1)\underline{\alpha}(t) \quad (87)$$

From which the following recursion on α is obtained

$$\underline{\alpha}(t+1) = \underline{\alpha}(t) + \underline{\underline{F}}^{-1}(t+1)\underline{\varepsilon}(t+1)\underline{D}(t+1) \quad (88)$$

where D is a diagonal matrix with elements

$$d_j(t+1) = P_{j+1}^{(t)} - 2P_j^{(t)} + P_{j-1}^{(t)} \quad (89)$$

and T is tridiagonal with elements

$$t_{j,j-1}(t+1) = -\frac{\Delta t}{4\Delta x^2}\left[P_{j+1}^{(t)} - P_{j-1}^{(t)}\right]\left[P_{j+1}^{(t)} - 2P_j^{(t)} + P_{j-1}^{(t)}\right] \quad (90)$$

$$t_{j,j}(t+1) = \frac{\Delta t}{\Delta x^2}\left[P_{j+1}^{(t)} - 2P_j^{(t)} + P_{j-1}^{(t)}\right]^2 \quad (91)$$

$$t_{j,j+1}(t+1) = -t_{j,j-1}(t+1) \quad (92)$$

The RLS procedures will be planned step by step for parameter estimation algorithm. In order to use RLS algorithms, the model needs to be re-written using a regressive form as compiled by [27], [28] and [29]:

$$y(t) = \varphi^T(t)\theta(t) + e(t) \quad (93)$$

Where φ^T is the transpose vector of regressive vector, φ (t); $\theta(t)$ is the vector of estimated parameters; $e(t)$is the sum of squared errors and $y(t)$is the output of the model.

Purpose of RLS algorithm is to obtain an estimate $\hat{\theta}(t)$at time t that minimizes the sum of squared errors of $e(t)$ based on measurements of the data vector $\varphi(t)$ and the output $y(t)$. At time t=0, we assume that $\hat{\theta}(t) = \theta_0$. Then at any time t, the estimate $\hat{\theta}(t)$ will be updated recursively according to RLS algorithm. The estimation is updated by adding a correction to the previous estimation at each step which is based on the error between the model and the outputs with the update gain. The RLS algorithm will be defined and adapted for estimating the parameters.

To understand recursive least squares, the least squares need to be estimated:

$$\hat{\theta}(t) = [X^T(t)X(t)]^{-1}X^T(t)Y(t) \tag{94}$$

With

$$Y^T(t) = [y(1) \ldots y(t) \tag{95}$$

$$X(t) = \begin{bmatrix} x^T(1) \\ . \\ . \\ . \\ x^T(t) \end{bmatrix} \tag{96}$$

Therefore, as one additional observation becomes available, the problem is then to find $\hat{\theta}(t+1)$ as a function of $\hat{\theta}(t)$and $y(t+1)$ and $u(t+1)$.

Defining $X(t+1)$ and $Y(t+1)$as

$$X(t+1) = \begin{bmatrix} X(t) \\ x^T(t+1) \end{bmatrix} \qquad Y(t+1) = \begin{bmatrix} Y(t) \\ y(t+1) \end{bmatrix} \tag{97}$$

And defining P(t) and P(t+1) as

$$P(t) = [X^T(t)X(t)]^{-1} \qquad P(t+1) = [X^T(t+1)X(t+1)]^{-1} \tag{98}$$

One can write

$$P(t+1) = [X^T(t)X(t) + x(t+1)x^T(t+1)]^{-1} \tag{99}$$

$$\hat{\theta}(t+1) = P(t+1)[X^T(t)Y(t) + x(t+1)y(t+1)] \tag{100}$$

Some simple matrix manipulations then give the recursive least-squares algorithm:

$$\hat{\theta}(t+1) = \hat{\theta}(t) + K(t+1)[y(t+1) - x^T(t+1)\hat{\theta}(t)] \tag{101}$$

$$K(t+1) = \frac{P(t)x(t+1)}{1 + x^T(t+1)P(t)x(t+1)} \tag{102}$$

$$P(t+1) = P(t) - \frac{P(t)x(t+1)x^T(t+1)P(t)}{1 + x^T(t+1)P(t)x(t+1)} \tag{103}$$

$$K(t+1) = P(t+1)x(t+1) \tag{104}$$

RLS algorithm:

$$\underbrace{\hat{\theta}(t+1)}_{new} = \underbrace{\hat{\theta}(t)}_{old} + \underbrace{K(t+1)}_{Kalman\ gain} \underbrace{[y(t+1) - x^T(t+1)\hat{\theta}(t)]}_{correction} \tag{105}$$

For recursive algorithm, data sequence will be dealt directly. Assume x(t+1)=0 for t+1<0, let $\hat{\theta}(t+1)$ be filter coefficient vector at time n and let $X\ (t+1)$ be input signal vector at time n.

$$\hat{\theta}(t+1) = \begin{bmatrix} \hat{\theta}(0,t) \\ \hat{\theta}(1,t+1) \\ \hat{\theta}(2,t+1) \\ . \\ . \\ . \\ \hat{\theta}(M-1t+1) \end{bmatrix} \qquad X(t+1) = \begin{bmatrix} x(t) \\ x(t+1) \\ x(\ t+2) \\ . \\ . \\ . \\ x(n-M+1) \end{bmatrix} \tag{106}$$

Step 1: Compute the filter output

$$\hat{d}(t+1) = x^T(t+1)\hat{\theta}(t) \tag{107}$$

Step 2: Compute the error

$$e_M(t+1) = y(t+1) - \hat{d}(t+1) \tag{108}$$

Step 3: Compute the Kalman gain vector:

$$K(t+1) = \frac{P(t)x(t+1)}{1+ x^T(t+1)P(t)x(t+1)} \tag{109}$$

Step 4: Update the inverse of the correlation matrix:

$$P(t+1) = P(t) - \frac{P(t)x(t+1)x^T(t+1)P(t)}{1+ x^T(t+1)P(t)x(t+1)} \tag{110}$$

Step 5: Update the coefficient vector of the filter:

$$\hat{\theta}(t+1) = \hat{\theta}(t) + K(t+1)\big[y(t+1) - x^T(t+1)\hat{\theta}(t)\big] \tag{111}$$

CHAPTER 5

PRINCIPAL COMPONENT ANALYSIS ALGORITHM

Among the large number of parameters, there are characteristic patterns and could be extracted out by means of a mathematical tool called Principal Component Analysis. The number of parameters to be optimized is reduced to the number of dominating geological patters present.

Optimization problem can be quite large if the unknown to be equal to the number of grid blocks. Therefore with PCA, number of parameters to be optimized is reduced to the number of dominating characteristics parameters only.

The PCA procedure for permeability reduction is as follows based on Yadav, S. [18]:

First, from the conceptual model, the set of permeability observed value for each grid block of the reservoir is obtained. Next, the covariance matrix of permeabilities will be calculated. The size of the covariance matrix would be m*m, where m is the number of grid blocks in the reservoir. The eigenvectors and the eigenvalues of the covariance matrix need to be calculated. In order to pick optimum number of parameters, we need to select the eigenvectors that is higher in eigenvalues as they described dominating patterns in the permeability data.

Method by Yadav,S. explained that the number of eigenvectors to be extracted can be obtained by dividing the corresponding eigenvalues with the trace of the covariance matrix. A threshold would be applied on the amount of total variance extracted.

Actually each eigenvector represent a characteristic pattern. However, in PCA method, only the dominating patterns are retained for analysis. This is due to the fact that a few eigenvectors can capture most of the information. Eigenvectors with low eigenvalues would not be included as it describes less important features.

$$y = V*x + z \qquad (112)$$

x = column vector containing weight of the dominating patterns

V = normalized eigenvectors of the covariance matrix,

$\overline{y}$ = column vector with mean of permeabilities at each grid block obtained from the set of permeability realizations

y = column vector containing new set of permeabilities with dominating patterns

$$\begin{bmatrix} z_1 \\ z_2 \\ . \\ . \\ z_m \end{bmatrix} = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ . & . \\ . & . \\ x_m & y_m \end{bmatrix} * \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} \overline{z}_1 \\ \overline{z}_2 \\ . \\ . \\ \overline{z}_m \end{bmatrix} \tag{113}$$

For preserving mean,

$$ax_i + by_i = z_i - \overline{z}_i \tag{114}$$

$$a \sum_{i=1}^{i=n} x_i + b \sum_{i=1}^{i=n} y_i = 0 \tag{115}$$

Where a and b are the value of weight of dominating patterns

For preserving variance,

$$ax_i + by_i + \overline{z}_i = z_i \tag{116}$$

$$(ax_i + by_i + \overline{z}_i)^2 = (z_i)^2 \tag{117}$$

$$(((ax_i + by_i + \overline{z}_i)^2)/n) - m^2 = ((z_i)^2/n) - m^2 \tag{118}$$

Where m= mean of permeability values of the realization

$$\sum_{i=1}^{i=n}(((ax_i + by_i + \overline{z}_i)^2)/n) - m^2 = Variance \tag{119}$$

Using,

$$\sum_{i=1}^{i=n} x_i^2 = \sum_{i=1}^{i=n} y_i^2 = 1, \text{(eigenvectors are unit vectors)} \tag{120}$$

$$\sum_{i=1}^{i=n} x_i * y_i = 0, \text{(eigenvectors are orthogonal)} \tag{121}$$

$$a^2 + b^2 = n * Variance + n * m^2 - \sum_{i=1}^{i=n} \overline{z}_i^2 - \sum_{i=1}^{i=n} 2 * (a * x_i + b * y_i) * \overline{z} \tag{122}$$

$$a^2 + b^2 + c_1 * a + c_2 * b = Constant \tag{123}$$

Where c_1 and c_2 are constants.

CHAPTER 6
PROJECT WORK

6.1 Research Methodology

6.1.1 Research and Review Literatures

After selection of topic, extensive research on history matching was done. Technical documents, journal articles, published papers are among extracted from few digital libraries. Among the library accessed includes OnePetro, Science Direct and IEEE explorer and also books from Information Resource Center, IRC UTP. These literatures were summarized, referred and also cited for the literature review and also for more understanding of the project.

6.1.2 Development of a Conceptual Model

The model was designed to be a simple arrangement of oil production well and gas injection well. The conceptual model portrays an area comprised of four gas injection well with an oil production well.

This work features a simple model setup for evaluating the applicability of combining a parameter reduction technique with a parameter estimation method for history matching. The purpose of the conceptual model is to produce two datasets of production performance which is determined by different values of parameter. Both generated data would be compared to see the differences between them.

6.1.3 Generation of synthetic historical data

One datasets would be taken as synthetic historical data since there are no realistic production history data available. For research purpose, the model is used to generate synthetic production history data. It will be later compared to the simulated data.

The objective of the simulation is to include forecasts of oil and water production rates. The performance of the model will be recorded for analysis. In this study, the output variable that will be compared is production data.

6.1.4 Generation of simulated data

Another datasets would be taken as the simulated data with the same method as the synthetic historical data.

6.1.5 Derivation of Forward Model & Definition of Objective Function

The forward model will be derived based on the literature as stated above in Chapter 4 and the objective function will be selected based on the best global objective function, simple least-squares formulation.

6.1.6 Analysis on RLS and PCA methods, applications and its successes

The RLS and PCA methods have both been applied for not only in history matching but also in other applications and the success was discussed to be further improved with the proposed algorithm of combination of RLS and PCA method.

6.1.7 Suggestion of flowchart for algorithm of combination of RLS and PCA

The flowchart will be designed to clarify step-by-step of applying RLS with addition of PCA after estimating initial parameters.

6.1.8 Application of RLS and PCA

The parameter estimation methods, RLS would be able to minimize the objective function with more accurate estimation of the parameters. The parameters will be run in the conceptual model and the discrepancy between historical data and new simulated data would be lesser than before.

Gantt chart for the whole project and key milestones are as Table 1 and Table 2 in Appendix A. The overall project flow is portrayed as in Figure 4 below.

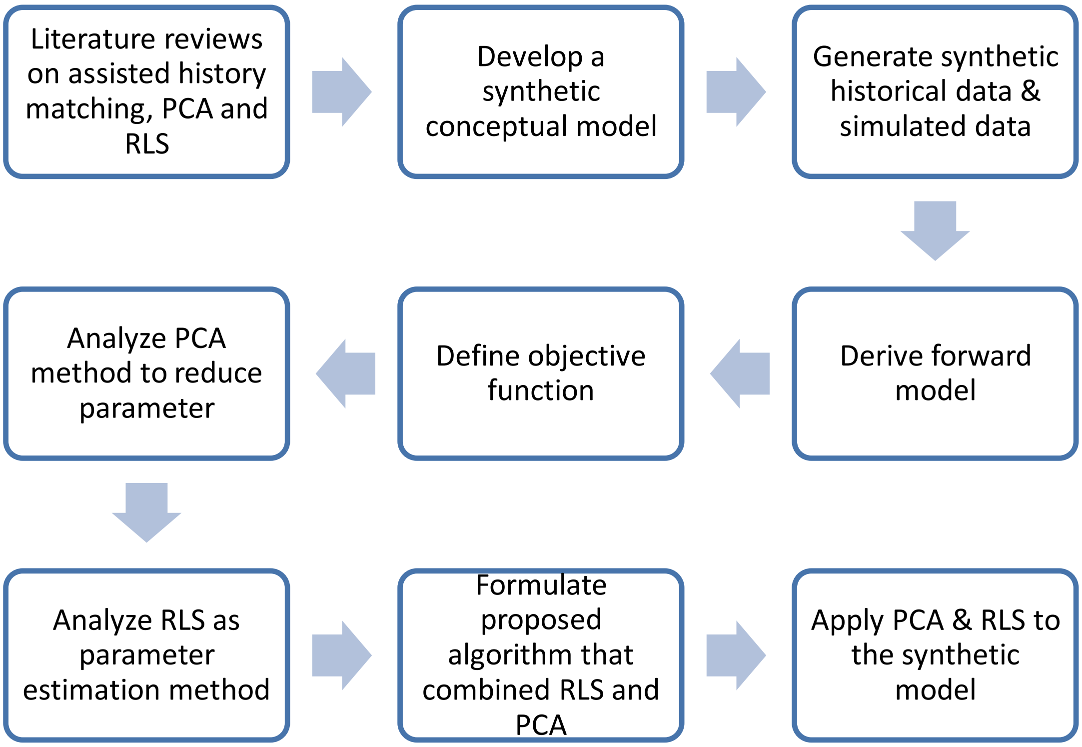

Figure 4 Flowchart for Project Methodology

6.2 Modified Synthetic Reservoir Model

A simple conceptual model of 3-dimensional with dimensions of 10 x 10 x 3 will be developed in a commercial reservoir simulator to portray features and behavior of a reservoir from a real field data. The bottomhole pressure is set to be constant at 1000 psia. In this study, the variables are limited to permeability only. The blocks are divided into 4 zones of different permeability each. The permeabilities are set to be different too at each Z layer. Therefore, total permeability values are limited to 12 different permeabilities for the model. The initial parameter of permeability for this model will be according to the zone. The porosity will be set as constant value of 30% to limit the parameters for the first case. There are 4 injection well; well Injection1 (INJ1), Injection2 (INJ2), Injection3 (INJ3) and Injection4 (INJ4) are located at coordinates of (1,1), (10,1), (10,10) and (1,10) respectively. The producing well (PROD) is located at the coordinate of (5,5), center of the reservoir. The conceptual model is planned to be constructed as shown in Figure 5 below.

The model that considered in this project contains three phases, oil, water and also gas. The thickness of z-direction layers differs, in which the first one is 20 ft, the second one is 30 ft and the last layer is 50 ft. The top layer is at 8325 ft. Both wells are completed and operated at fixed bottomhole pressures. The well depth is between 8325 ft to 8425 ft.

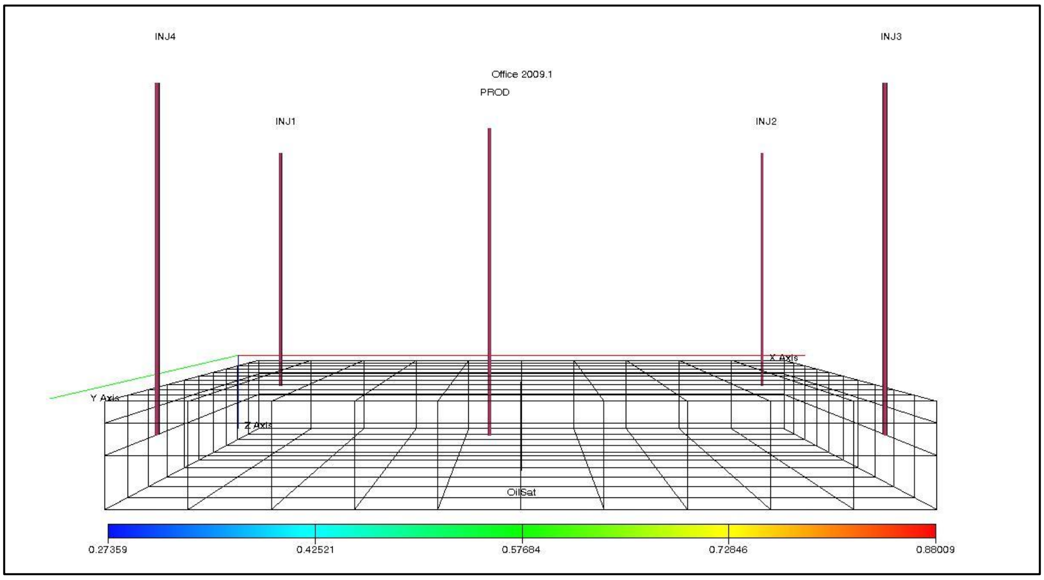

Figure 5 Side 3-D view of the model

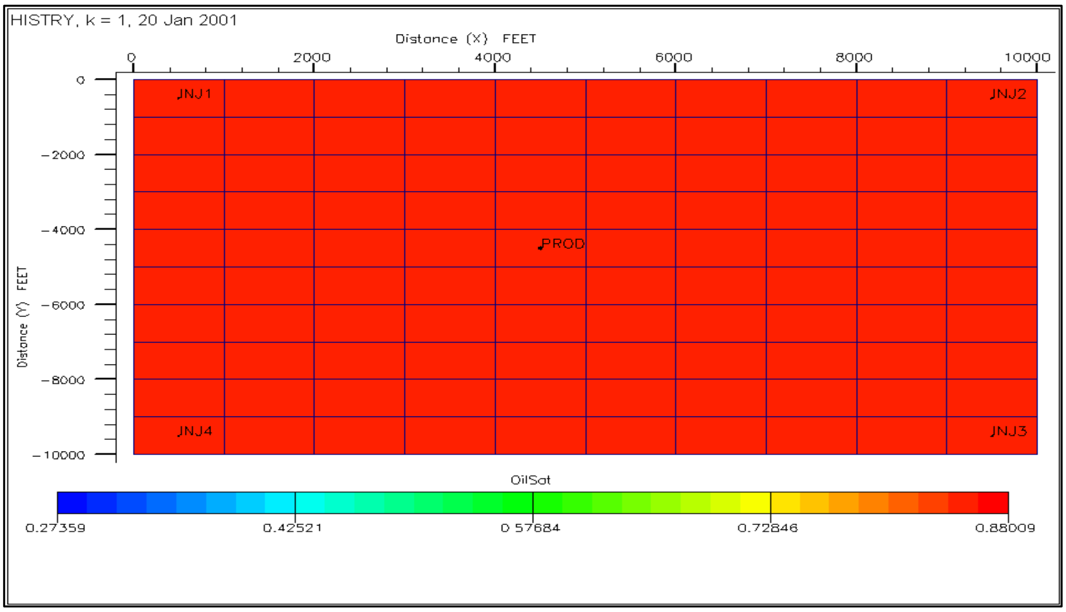

Figure 6 Top 2-D View of the model

Each injection well has been set to inject 50000 Mscf of gas while the control rate for production well is to be constant BHP pressure of 1000 psia.

The simulation was then run, using ECLIPSE simulator for 6 years with three quantities (gas production rate, Qgp, oil production rate, Qop, and gas injection rate, Qgi) being recorded at the end of each month.

CHAPTER 7
RESULTS AND DISCUSSION

7.1 Simulation

The implementation of the methods was decided to be tested using conceptual model. The conceptual model was built using a commercial reservoir simulator that represents the characteristics and the behavior of a reservoir. With all the data presents, the synthetic historical data is obtained and comparing it with the simulated data that is also obtained from the conceptual model.

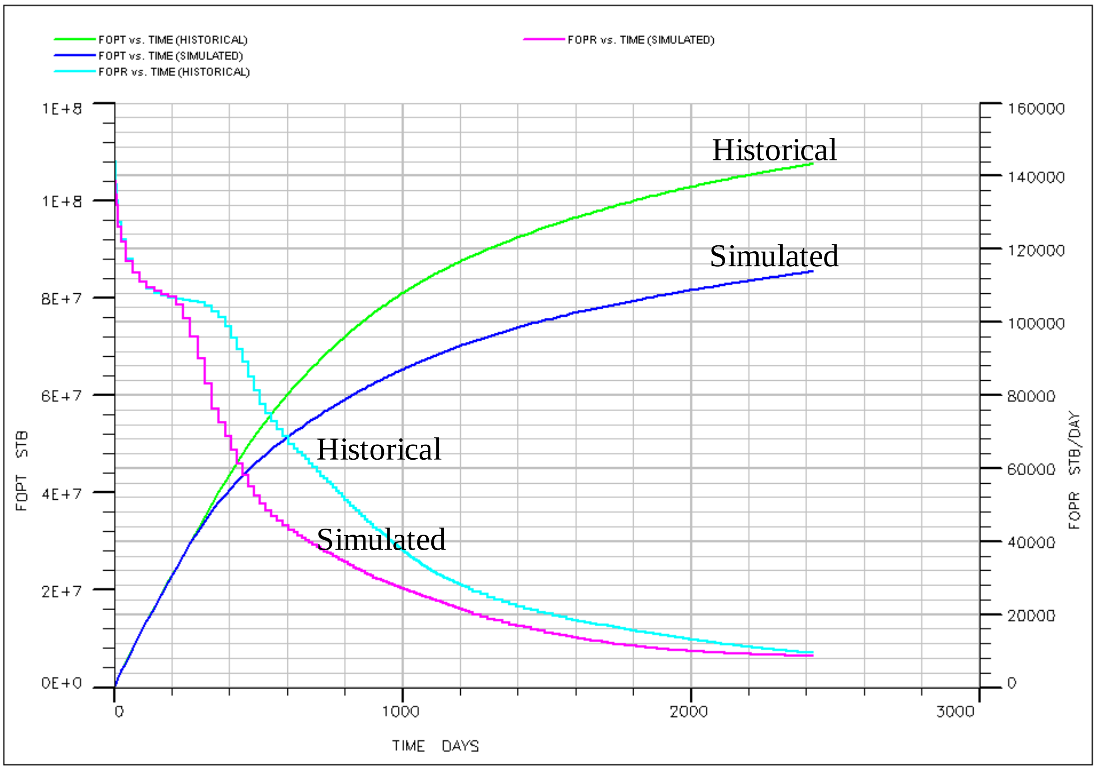

Figure 7 Comparison of Historical and Simulated data

From the figure shown above, it can be seen that there are some discrepancy among the historical data and the simulated. The misfits between these two data are the one to be minimized. If simulated data's performance almost similar to the historical data, it shows that the simulation succeeding in matching the reservoir characterization.

Other criteria that can be compared with for the behavior of simulated and historical data, is for GOR data. GOR stands for gas-oil ratio, in which the ratio of produced gas with production of 1 barrel of oil. This property can also be compared same as production rate.

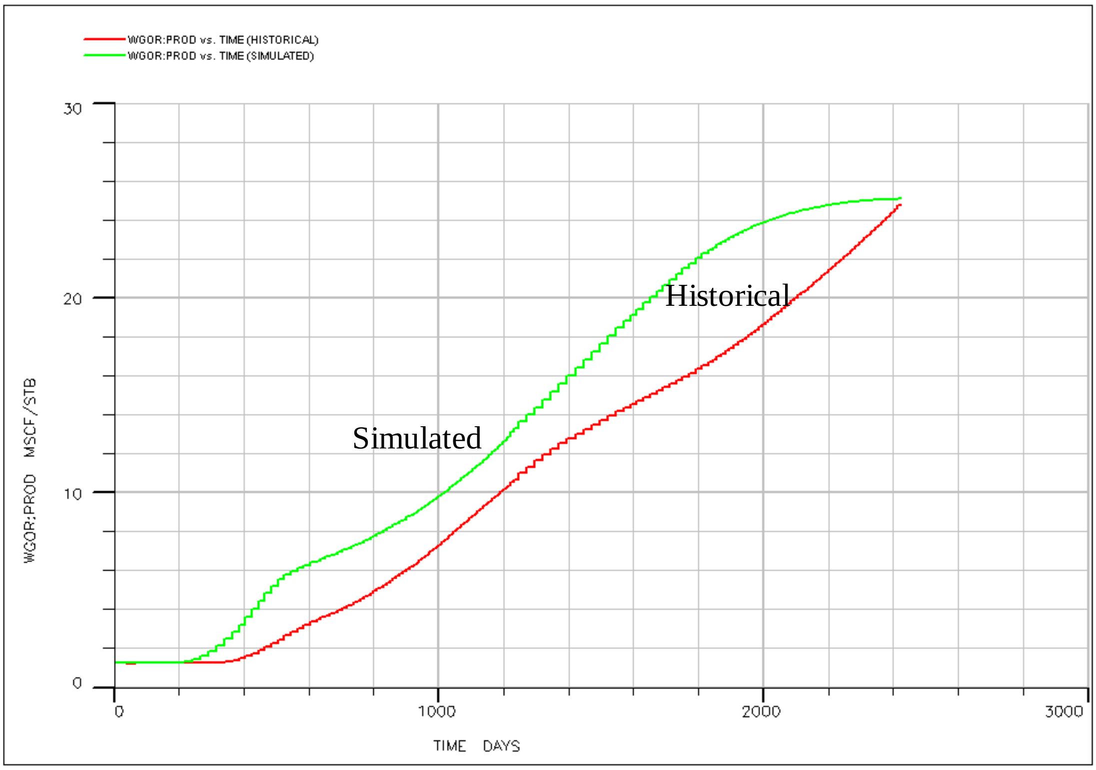

Figure 8 Comparison of GOR between historical and simulated

7.2 RLS and Its Successes in Estimating Parameters

A study was conducted by Murad, F. [26], of using recursive least squares method for estimation of reservoir parameters. Based on the study, the pressure and temperature responses were observed and matched in history matching by identifying reservoir parameters such as permeability. The method has been proven to work as the convergence is fast and managed to be estimated to accuracy up to 10^{-6}. This shows that with good initial estimates, the method can accurately identify the correct estimation. Based on the study, it was observed that RLS produced unstable results with higher number of parameters. This was concluded in the study as the finite difference scheme's stability depends on the choice of the time step and distance step as well as on the initial estimates of the unknown parameters.

Another issue arises with RLS method is even with rapid convergence, the correction term to the parameter being estimated was large. In some example, it can be almost to the order of 10^{200}.

Recursive Least Squares (RLS) Algorithm

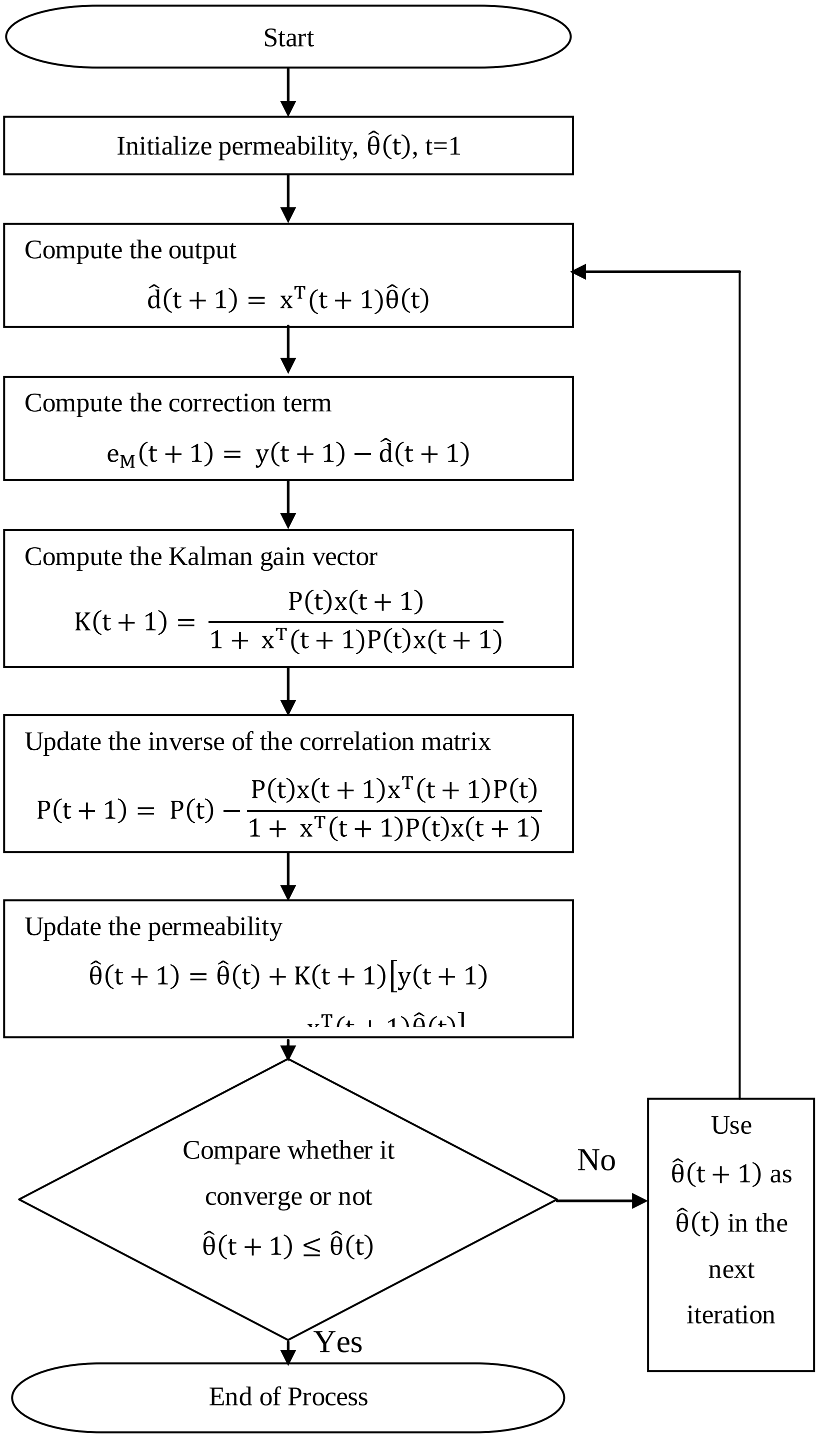

Figure 9 Flowchart of RLS algorithm

From the study of RLS method, a flowchart with the RLS algorithm is formulated. The process is recursive until it converges with the true parameter.

7.3 Flowchart of Combined RLS and PCA algorithm

The RLS algorithm was arranged to be applied in MATLAB code as a straightforward implementation of the RLS algorithm.

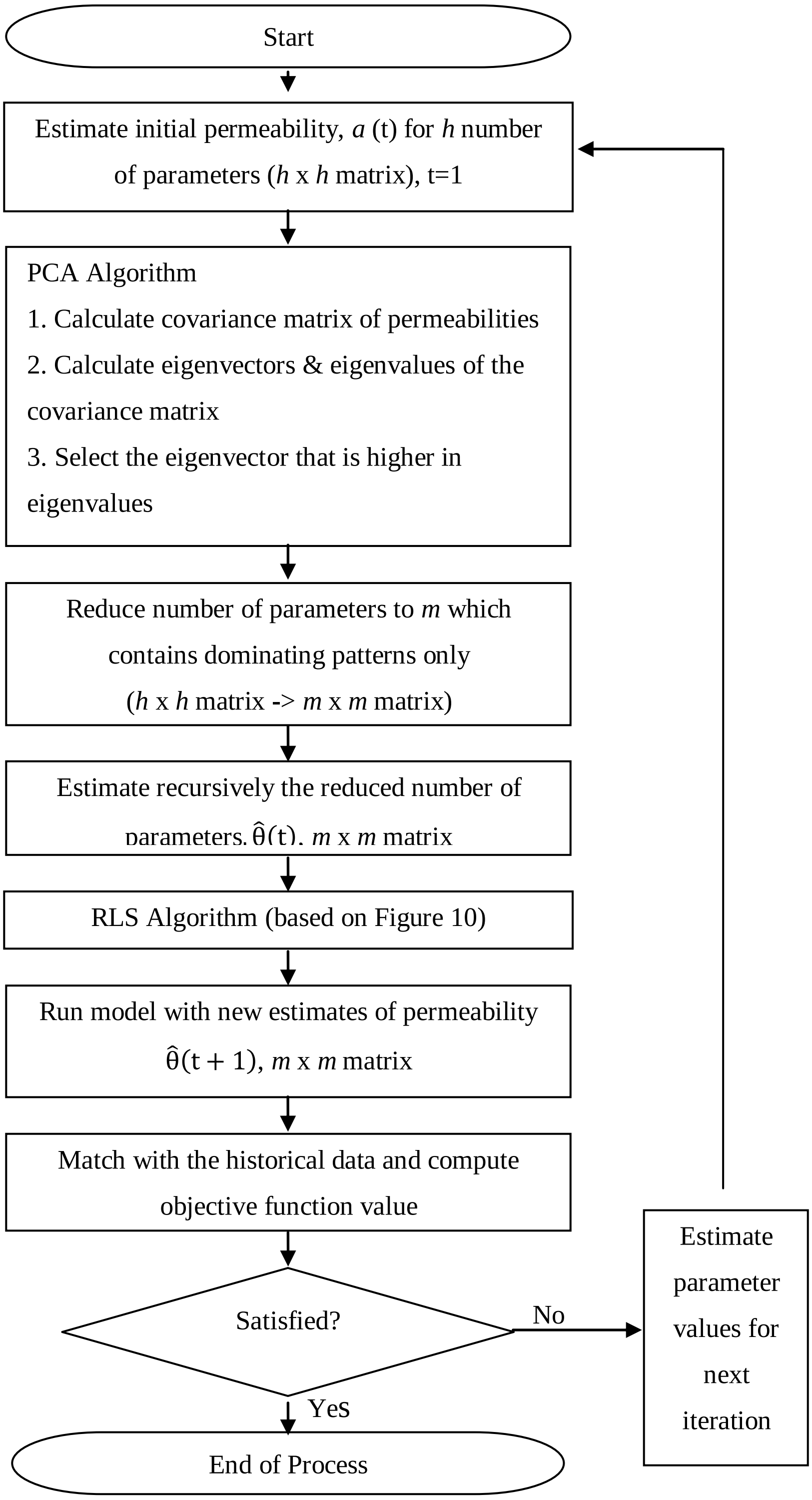

Figure 10 Flowchart of Combined PCA & RLS algorithm

Applying RLS & PCA

Recursive means continuing updating with consideration of the previous data. It means when we have initial data and we apply the RLS method, the next iteration will also consider the first timestep and this will be updating continuously with the gain.

The next step would be applying for RLS and PCA using a numerical computational tool. One way of understanding the methods is by tracing the conditions; parameters needed and expected output for the following method using ready-made examples. Examples obtained from available numerical computational tool code helps in understanding ways to apply the methods with available data.

Production rate will be extracted from the simulation and these data serves as the observed data and initial guess of the permeability would be randomly guessed to be converged using RLS method. After applying RLS and PCA, analysis of the results can be obtained and compared the updated simulated data with the historical data.

7.4 Matched simulation

After application of RLS and PCA, the new estimated permeability is used as input in ECLIPSE reservoir simulator and run once again. The result as in figure below shows that the new estimated parameter shows better result compared to the initial simulation case.

Other production parameters such as GOR can also be seen showing similar trends between historical and the new matched result. The new estimated permeability could be very close to the true parameters.

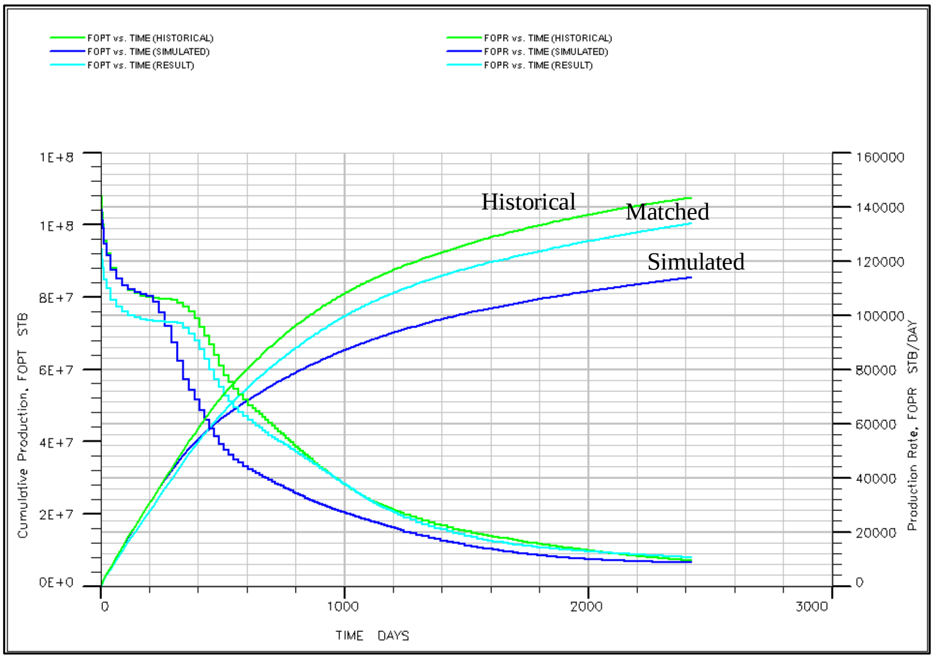

Figure 11 Comparison of Production Rate of Historical, simulated and matched data

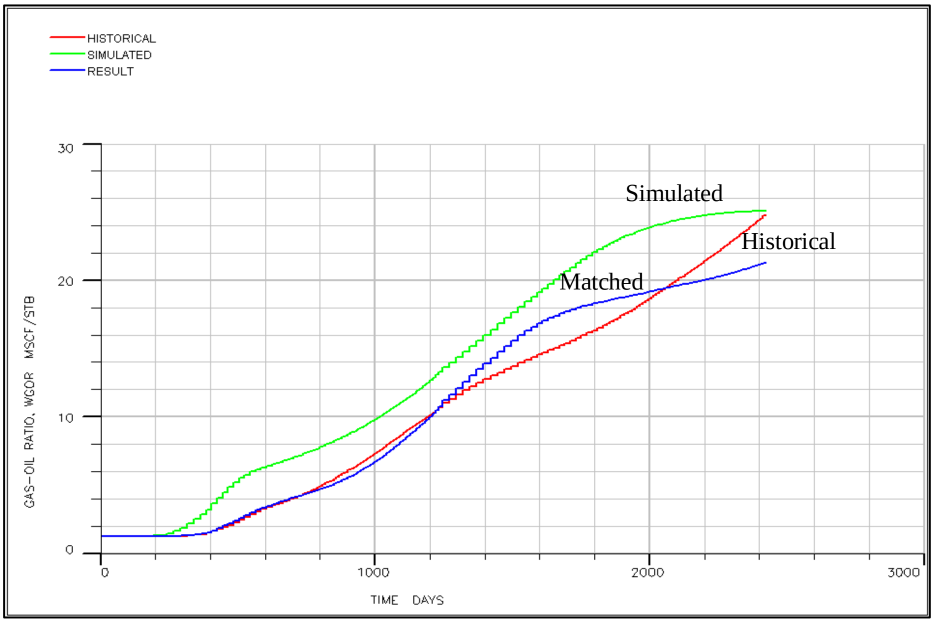

Figure 12 Comparison of GOR between Historical, simulated and matched data

Another comparison in terms of mean-square error was also done to prove the efficiency of RLS and PCA method.

Mean-Square Error, % =

$$\frac{\sum[(Hist.Q - \ Sim.Q)^2]}{\sum(Hist.Q)^2} \times 100\%$$

$$\frac{\sum[(Hist.Q - \ Match.Q)^2]}{\sum(Hist.Q)^2} \times 100\%$$

After calculating, the error between historical and initial simulated data results in 5.17%. Meanwhile the error between historical and new matched results is only 0.75%. This proves that when combined application of RLS and PCA is applied, almost 5% of error can be reduced and thus more accurate. This is important in order to accurately predicting the future performance of the reservoir.

7.5 Analysis on Oil Saturation Distribution

Basically, when the process of history matching is done correctly that it able to replicate the behavior of the reservoir, then the performance of the model can be analyzed as it shows and predicts the reservoir performance. The analysis such as oil saturation and pressure distribution can detect for unswept region or pressure profiles respectively thus can design for pressure maintenance plan, enhanced oil recovery or other alternatives. In this study, oil saturation distribution analysis was done and shown in the following discussion.

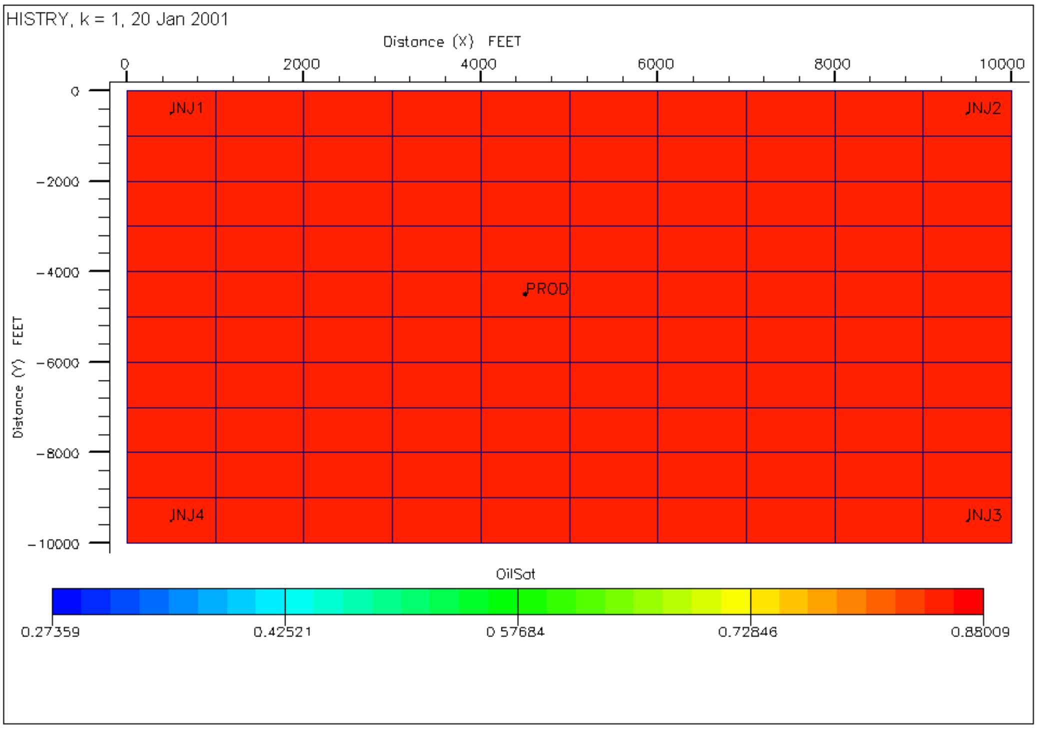

Figure 13 2-D Oil Saturation (Day 1)

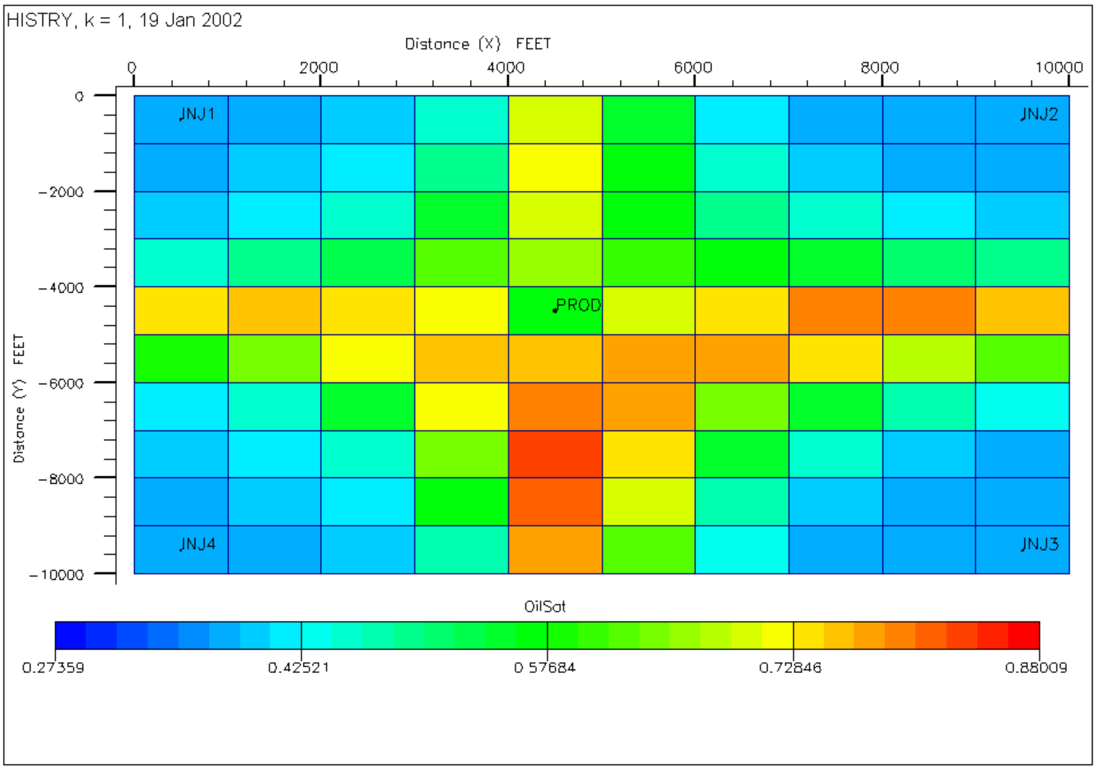

Figure 14 2-D Oil Saturation (Day 365)

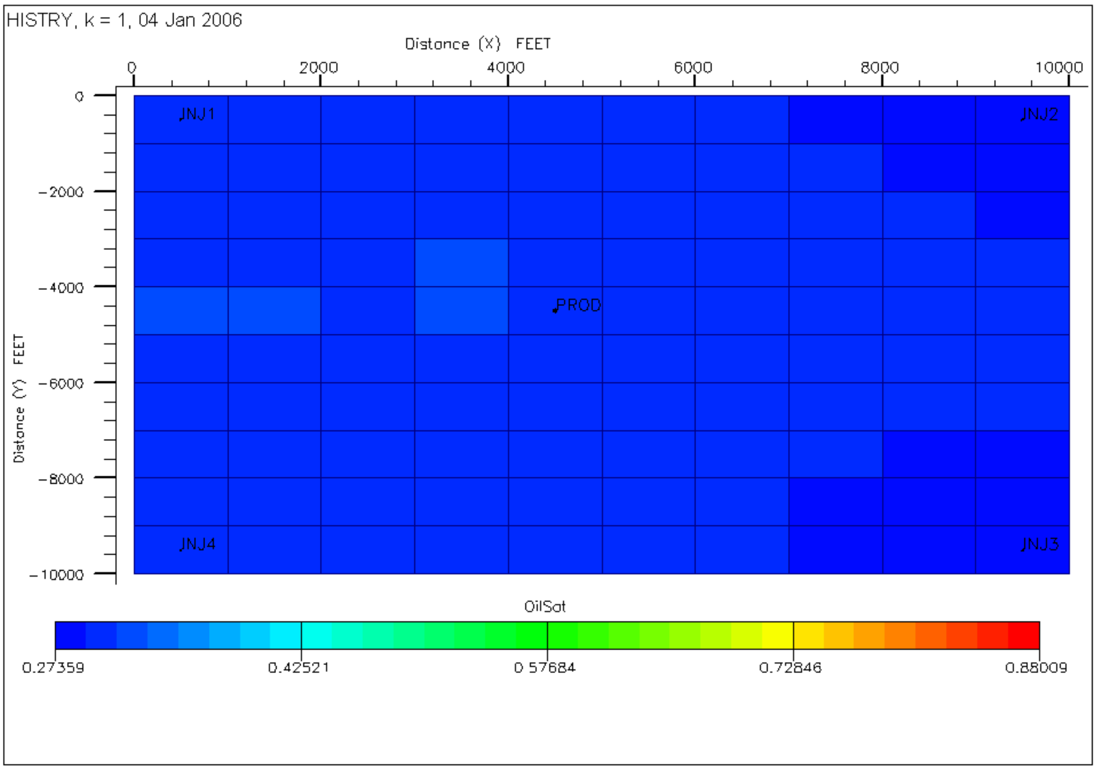

Figure 15 2-D Oil Saturation (Day 900)

From the reservoir simulator, oil saturation distribution can be seen for every block for different day or period. Figure 13, Figure 14 and Figure 15 shows the oil saturation distribution for 300 blocks at day 1, day 300 and day 900. The difference can be seen at as the oil is produced and results in decrease in oil saturation.

With history matching, all the production performance of the well can be forecasted and thus helps in decision-making process in order to fully extract the sources with less expense. Several scenarios can be designed in the simulation and the performance can be directly evaluated by the reservoir engineers as soon as the history matching is accurate. Therefore, with less time taken to match the historical data with the simulated data, more profits can be obtained.

CHAPTER 8
CONCLUSIONS & RECOMMENDATIONS

8.1 Conclusions

The project aim's is to suggest an alternative in optimization methods for better results of assisted history matching using a combination of parameter estimation technique and parameter reduction methods. Assisted history matching serves as technique to reduce the time taken for history matching process for better reservoir management.

In this study the focus is on combination of RLS, for parameter estimation purpose, with PCA, for parameter reduction purpose, in order to solve history matching problem. Various literature reviews have been explained to show the application of both methods in petroleum or other industry's purposes. RLS method has not been widely used in history matching due to unable to stabilize for higher number of parameters. In this project, the RLS method is suggested to be run together with parameter reduction method, PCA method. As the project progresses, it is decided to formulate for the algorithm that combined both methods. The application of the methods was decided for a synthetic model for simpler purpose. The synthetic model was built in order to represent the synthetic historical data of the reservoir.

The forward model was also derived in order to relate between the input parameters with the expected output. Based on other field application, RLS can therefore be applied for assisted history matching when combined with PCA. An algorithm of combination of PCA & RLS is formulated and therefore can be applied in a synthetic reservoir. The algorithm is ready to be used provided with ample time, knowledge and skills in handling the numerical computational tool. The findings in this project are able to show firm stands regarding the applicability of the methods. Last but not least, the findings can be used as the basic concept to be further implemented on a more complex reservoir in order to save the time taken for manual history matching.

8.2 Recommendations

Due to several limitations, a few recommendations are suggested to be used in further studies.

a) RLS need to be evaluated under more realistic conditions such as for large-scale reservoir models.
b) Proposed algorithm need to be improved and refined in order to be practically applied for industrial purpose.

REFERENCES

1. Satter, A., J.E. Varnon, and M.T. Hoang, *Integrated Reservoir Management.* 1992.

2. Jacquard, P., *Permeability Distribution From Field Pressure Data.* 1965.

3. Gao, G., G. Li, and A.C. Reynolds, *A Stochastic Optimization Algorithm for Automatic History Matching*, 2004, Society of Petroleum Engineers.

4. Cancelliere, M., F. Verga, and D. Viberti, *Benefits and Limitations of Assisted History Matching*, 2011, Society of Petroleum Engineers.

5. Rwechungura, R.W., M. Dadashpour, and J. Kleppe, *Advanced History Matching Techniques Reviewed*, 2011, Society of Petroleum Engineers.

6. Oliver, D.S., Albert C. Reynolds, and N. Liu., *Inverse Theory for Petroleum Reservoir Characterization and History Matching*. 2008: Cambridge University Press.

7. Carter, J.N., *Using Bayesian Statistics to Capture the Effects of Modelling Errors in Inverse Problems.* Mathematical Geology, 2004. 36(No. 2).

8. Ardalan, S.H., *Floating-point error analysis of recursive least-squares and least-mean-squares adaptive filters.* Circuits and Systems, IEEE Transactions on, 1986. 33(12): p. 1192-1208.

9. Cioffi, J.M., *Limited-precision effects in adaptive filtering.* Circuits and Systems, IEEE Transactions on, 1987. 34(7): p. 821-833.

10. Mattax, C.C. and R.L. Dalton, *Reservoir Simulation*. 1990: Henry L. Doherty Memorial Fund of AIME, Society of Petroleum Engineers.

11. Mata-Lima, H., *Evaluation of the objective functions to improve production history matching performance based on fluid flow behaviour in reservoirs.* Journal of Petroleum Science and Engineering, 2011. 78(1): p. 42-53.

12. Bertolini, A.C. and D.J. Schiozer, *Influence of the objective function in the history matching process.* Journal of Petroleum Science and Engineering, 2011. 78(1): p. 32-41.

13. Tillier, E., S.D. Veiga, and R. Derfoul, *Appropriate formulation of the objective function for the history matching of seismic attributes.* Computers & Geosciences, 2013. 51: p. 64-73.

14. Plackett, R.L., *Some Theorems in Least Squares.* 1950.

15. Zarchan, P. and H. Musoff, *Fundamentals of Kalman Filtering: A Practical Approach*, ed. F.K. Lu. Vol. 232. 2009: American Institute of Aeronautics and Astronautics, Inc. 883.

16. Mitsis, G.D. and M.M. Markou. *Recursive least squares estimation of nonlinear multiple-input systems using orthonormal function expansions*. in *Engineering in Medicine and Biology Society,EMBC, 2011 Annual International Conference of the IEEE*. 2011.

17. Wang, C. and T. Tang, *Recursive least squares estimation algorithm applied to a class of linear-in-parameters output error moving average systems.* Applied Mathematics Letters, 2014. 29(0): p. 36-41.

18. Yadav, S., *History Matching Using Face-Recognition Technique Based on Principal Component Analysis*, 2006, Society of Petroleum Engineers.

19. Huang, X., L.R. Bentley, and C. Laflamme, *Seismic History Matching Guided By Attribute Zonation*, 2001, Society of Exploration Geophysicists.

20. Scheevel, J.R. and K. Payrazyan, *Principal Component Analysis Applied to 3D Seismic Data for Reservoir Property Estimation.* 2001.

21. Sarma, P., et al., *A New Approach to Automatic History Matching Using Kernel PCA*, 2007, Society of Petroleum Engineers.

22. Sarma, P., L.J. Durlofsky, and K. Aziz, *Kernel PCA for Efficient, Differentiable Parameterization of Multipoint Geostatistics.* Math Geosci, 2007. 40 p. 3-32.

23. M. Khaninezhad, M. and B. Jafarpour, *Hybrid Parameterization for Robust History Matching.*

24. Kleppe, J., *Three Phase Flow.* TPG4160 Reservoir Simulation 2011, 2011.

25. Chen, W.H., et al., *A new algorithm for automatic history matching* 1973.

26. Murad, F., *Recursive Least Square Methods for Estimation of Reservoir Parameters.* 1998.

27. Jayaraman, G.P. and S.V. Lunzman. *Parameter estimation of an electronic load sensing pump using the Recursive Least Squares algorithm*. in *Decision and Control (CDC), 2010 49th IEEE Conference on*. 2010.

28. Huynh, D.C., M.W. Dunnigan, and S.J. Finney. *On-line parameter estimation of an induction machine using a recursive least-squares algorithm with multiple time-varying forgetting factors*. in *Power and Energy (PECon), 2010 IEEE International Conference on*. 2010.

29. Huixin, C. *Extended recursive least squares algorithm for nonlinear stochastic systems*. in *American Control Conference, 2004. Proceedings of the 2004*. 2004.

APPENDICES

Appendix A: Gantt chart

Gantt chart is presented as Table 1 to represent activities in FYP 1.

Table 1 Gantt chart for FYP 1

Detail / Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	FYP 1 (January – April)													
Selection of Project Topic														
Research Work / Literature review														●
Data Gathering										●				
Development of conceptual model													●	
Generation of simulated data														●
Derivation of forward model														●
Preparation of Interim Report														●

Gantt chart is presented as Table 2 to represent activities in FYP 2.

Table 2 Gantt chart for FYP 2

Detail / Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	FYP 2 (May – September)														
Literature review											●				
Examples for RLS and PCA		●													
Objective function					●										
Analysis of RLS & PCA															
Proposal of algorithm - Combination of RLS & PCA									●						
Submission of Progress Report							▲								
Analysis and Comparison of Results									●						
Pre- Sedex										▲					
Preparation of Draft Final Report											●				
Submission of Draft Final Report											▲				
Submission of Dissertation (Soft)												▲			
Submission of Technical Paper												▲			
Viva													▲	▲	
Submission of Dissertation (Hard)															▲

Appendix B: Key Milestones

Key milestones for FYP 1

- Research study regarding Assisted history matching, Principal Component Analysis (PCA) and Recursive Least Squares (RLS) algorithm
- Finalized data for conceptual model
- Developed the conceptual model to be used for simulation
- Generated the simulated data and compared with the historical data
- Derived the forward model to obtain the generalized formula
- Finished the Interim Report as final submission for FYP 1

Key milestones for FYP 2

- Research study regarding Assisted history matching, Principal Component Analysis (PCA) and Recursive Least Squares (RLS) algorithm
- Understand the concept of RLS and PCA with examples
- Come up with the objective function for the particular method
- Come up with algorithm of combined RLS and PCA for its theoretical application and successes.
- Discussion of the results
- Finished the Final Report as final submission for FYP 2

Appendix C: Step-by-Step Derivation of Forward Model

A discrete set of grid block in the x and y plane is obtained by the use of a grid system to divide the solution rectangle.

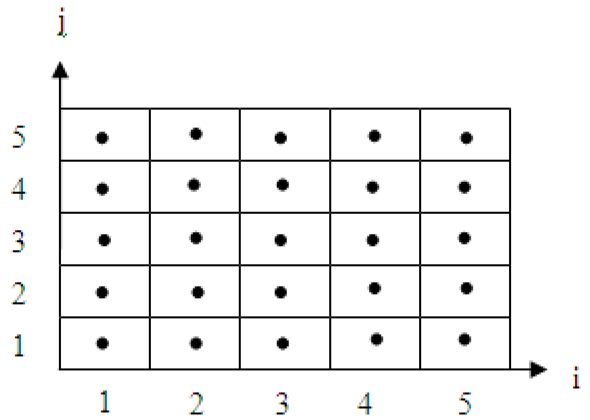

Figure 16 Gridblocks in X-Y Plane

There are i gridblocks in the x-direction and j gridblocks in the y-direction. Point (x_i, y_i) lies at the center of a gridblock.

Locations of the gridblocks are defined by locations of their boundaries

$x_{1/2}, x_{3/2}, \dots x_{i+1/2}$ $\qquad\qquad$ $y_{1/2}, y_{3/2}, \dots y_{j+1/2}$

The centers of the gridblocks

$x_i = \frac{(x_{i+1/2} + x_{i-1/2})}{2}$ $\qquad\qquad$ $y_i = \frac{(y_{j+1/2} + y_{j-1/2})}{2}$

Oil phase

$$\frac{\partial}{\partial x}\left(k_x \lambda_o \frac{\partial P_O}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \lambda_o \frac{\partial P_O}{\partial y}\right) = \frac{\partial}{\partial t}(\emptyset \frac{S_o}{B_O})$$

Water phase

$$\frac{\partial}{\partial x}\left(k_x \lambda_w \frac{\partial P_w}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \lambda_w \frac{\partial P_w}{\partial y}\right) = \frac{\partial}{\partial t}(\emptyset \frac{S_w}{B_w})$$

Spatial Discretization

Oil phase

X-dimensional

$$\frac{\partial}{\partial x}\left(k_x\lambda_o\frac{\partial P_O}{\partial x}\right)\sim\frac{\frac{(k_x\lambda_o)_{i+1/2,j}\left[(P_o)_{i+1,j}-(P_o)_{i,j}\right]}{x_{i+1}-x_i}-\frac{(k_x\lambda_o)_{i-1/2,j}\left[(P_o)_{i,j}-(P_o)_{i-1,j}\right]}{x_i-x_{i-1}}}{x_{i+1/2}-x_{i-1/2}}$$

Y-dimensional

$$\frac{\partial}{\partial y}\left(k_y\lambda_o\frac{\partial P_O}{\partial y}\right)\sim\frac{\frac{(k_y\lambda_o)_{i,j+1/2}\left[(P_o)_{i,j+1}-(P_o)_{i,j}\right]}{y_{j+1}-y_j}-\frac{(k_y\lambda_o)_{i,j-1/2}\left[(P_o)_{i,j}-(P_o)_{i,j-1}\right]}{y_j-y_{j-1}}}{y_{j+1/2}-y_{j-1/2}}$$

Water phase

X-dimensional

$$\frac{\partial}{\partial x}\left(k_x\lambda_w\frac{\partial P_w}{\partial x}\right)\sim\frac{\frac{(k_x\lambda_w)_{i+1/2,j}\left[(P_w)_{i+1,j}-(P_w)_{i,j}\right]}{x_{i+1}-x_i}-\frac{(k_x\lambda_w)_{i-1/2,j}\left[(P_w)_{i,j}-(P_w)_{i-1,j}\right]}{x_i-x_{i-1}}}{x_{i+1/2}-x_{i-1/2}}$$

Y-dimensional

$$\frac{\partial}{\partial y}\left(k_y\lambda_w\frac{\partial P_O}{\partial y}\right)\sim\frac{\frac{(k_y\lambda_w)_{i,j+1/2}\left[(P_w)_{i,j+1}-(P_w)_{i,j}\right]}{y_{j+1}-y_j}-\frac{(k_y\lambda_w)_{i,j-1/2}\left[(P_w)_{i,j}-(P_w)_{i,j-1}\right]}{y_j-y_{j-1}}}{y_{j+1/2}-y_{j-1/2}}$$

Time Discretization

Oil phase

$$\frac{\partial}{\partial t}\left(\emptyset\frac{S_o}{B_O}\right)\sim\frac{\emptyset_{i,j}(1-S_{w_{i,j}})}{\Delta t}\left[\frac{C_r}{B_O}+\frac{\partial(1/B_O)}{\partial P_O}\right]\left[(P_o)_{i,j}^{t+1}-(P_o)_{i,j}^{t}\right]-\left[\frac{\emptyset}{B_O\Delta t}\right]_{i,j}\left[(S_w)_{i,j}^{t+1}-(S_w)_{i,j}^{t}\right]$$

Water phase

$$\frac{\partial}{\partial t}\left(\emptyset\frac{S_w}{B_w}\right)\sim\frac{\emptyset_{i,j}(S_{w_{i,j}})}{\Delta t}\left[\frac{C_r}{B_w}+\frac{\partial(1/B_w)}{\partial P_w}\right]\left[(P_o)_{i,j}^{t+1}-(P_o)_{i,j}^{t}\right]-\left[\left[\frac{\emptyset}{B_O\Delta t}\right]_{i,j}-\left(\frac{dP_{cow}}{dS_w}\right)\left(\frac{S_{w_{i,j}}\emptyset_{i,j}}{\Delta t}\left[\frac{C_r}{B_w}+\frac{\partial(1/B_w)}{\partial P_w}\right]\right)\right]\left[(S_w)_{i,j}^{t+1}-(S_w)_{i,j}^{t}\right]$$

Basic Finite-Difference Equation

Substitution of approximation for the spatial and time Discretization into differential equation

Oil phase

$$\frac{\frac{(k_x\lambda_o)_{i+1/2,j}\left[(P_o)_{i+1,j}-(P_o)_{i,j}\right]}{x_{i+1}-x_i}-\frac{(k_x\lambda_o)_{i-1/2,j}\left[(P_o)_{i,j}-(P_o)_{i-1,j}\right]}{x_i-x_{i-1}}}{x_{i+1/2}-x_{i-1/2}}$$

$$+\frac{\frac{(k_y\lambda_o)_{i,j+1/2}\left[(P_o)_{i,j+1}-(P_o)_{i,j}\right]}{y_{j+1}-y_j}-\frac{(k_y\lambda_o)_{i,j-1/2}\left[(P_o)_{i,j}-(P_o)_{i,j-1}\right]}{y_j-y_{j-1}}}{y_{j+1/2}-y_{j-1/2}}$$

$$=\frac{\emptyset_{i,j}(1-S_{w_{i,j}})}{\Delta t}\left[\frac{C_r}{B_O}+\frac{\partial(1/B_O)}{\partial P_O}\right]\left[(P_o)_{i,j}^{t+1}-(P_o)_{i,j}^{t}\right]$$

$$-\left[\frac{\emptyset}{B_O\Delta t}\right]_{i,j}\left[(S_w)_{i,j}^{t+1}-(S_w)_{i,j}^{t}\right]$$

Water phase

$$\frac{\frac{(k_x\lambda_w)_{i+1/2,j}\left[(P_w)_{i+1,j}-(P_w)_{i,j}\right]}{x_{i+1}-x_i}-\frac{(k_x\lambda_w)_{i-1/2,j}\left[(P_w)_{i,j}-(P_w)_{i-1,j}\right]}{x_i-x_{i-1}}}{x_{i+1/2}-x_{i-1/2}}$$

$$+$$

$$\frac{\frac{(k_y\lambda_w)_{i,j+1/2}\left[(P_w)_{i,j+1}-(P_w)_{i,j}\right]}{y_{j+1}-y_j}-\frac{(k_y\lambda_w)_{i,j-1/2}\left[(P_w)_{i,j}-(P_w)_{i,j-1}\right]}{y_j-y_{j-1}}}{y_{j+1/2}-y_{j-1/2}}$$

$$= \frac{\emptyset_{i,j}(S_{w_{i,j}})}{\Delta t}\left[\frac{C_r}{B_w}+\frac{\partial(1/B_w)}{\partial P_w}\right]\left[(P_o)_{i,j}^{t+1}-(P_o)_{i,j}^{t}\right]-\left[\left[\frac{\emptyset}{B_O\Delta t}\right]_{i,j}-\left(\frac{dP_{cow}}{dS_w}\right)\left(\frac{S_{w_{i,j}}\emptyset_{i,j}}{\Delta t}\left[\frac{C_r}{B_w}+\frac{\partial(1/B_w)}{\partial P_w}\right]\right)\right]\left[(S_w)_{i,j}^{t+1}-(S_w)_{i,j}^{t}\right]$$

Transmissibilities

$$(T_{x_l})_{i+1/2,j} = \frac{(k_x\lambda_l)_{i+1/2,j}}{x_{i+1}-x_i} = \frac{2(\lambda_l)_{i+1/2,j}}{\frac{\Delta x_{i+1}}{k_{i+1}}+\frac{\Delta x_i}{k_i}}$$

$$(T_{x_l})_{i-1/2,j} = \frac{(k_x\lambda_l)_{i-1/2,j}}{x_i-x_{i-1}} = \frac{2(\lambda_l)_{i-1/2,j}}{\frac{\Delta x_i}{k_i}+\frac{\Delta x_{i-1}}{k_{i-1}}}$$

$$(T_{y_l})_{i,j+1/2} = \frac{(k_y\lambda_l)_{i,j+1/2}}{y_{i+1}-y_i} = \frac{2(\lambda_l)_{i,j+1/2}}{\frac{\Delta y_{i+1}}{k_{i+1}}+\frac{\Delta y_i}{k_i}}$$

$$(T_{y_l})_{i,j-1/2} = \frac{(k_y\lambda_l)_{i,j-1/2}}{y_i-y_{i-1}} = \frac{2(\lambda_l)_{i,j-1/2}}{\frac{\Delta y_i}{k_i}+\frac{\Delta y_{i-1}}{k_{i-1}}}$$

Upstream mobilities

$\lambda_{l_{i+1/2,j}}$ = $[\lambda_{l_{i+1,j}}$ if $(P_l)_{i+1,j} \geq (P_l)_{i,j}]$ or $[\lambda_{l_{i,j}}$ if $(P_l)_{i+1,j} < (P_l)_{i,j}]$

$\lambda_{l_{i+1/2,j}}$ = $[\lambda_{l_{i-1,j}}$ if $(P_l)_{i-1,j} \geq (P_l)_{i,j}]$ or $[\lambda_{l_{i,j}}$ if $(P_l)_{i-1,j} < (P_l)_{i,j}]$

$\lambda_{l_{i,j+1/2}}$ = $[\lambda_{l_{i,j+1}}$ if $(P_l)_{i,j+1} \geq (P_l)_{i,j}]$ or $[\lambda_{l_{i,j}}$ if $(P_l)_{i,j+1} < (P_l)_{i,j}]$

$\lambda_{l_{i,j-1/2}}$ = $[\lambda_{l_{i,j-1}}$ if $(P_l)_{i,j-1} \geq (P_l)_{i,j}]$ or $[\lambda_{l_{i,j}}$ if $(P_l)_{i,j-1} < (P_l)_{i,j}]$ l = o,w

Right side coefficients

$$C_{poo_{i,j}} = \frac{\emptyset_{i,j}(1-S_{w_{i,j}})}{\Delta t}\left[\frac{C_r}{B_O}+\frac{\partial(1/B_O)}{\partial P_O}\right]_{i,j}$$

$$C_{swo_{i,j}} = -\frac{\emptyset_{i,j}}{B_{o_{i,j}}\Delta t_{i,j}}$$

$$C_{pow_{i,j}} = \frac{\emptyset_{i,j}(S_{w_{i,j}})}{\Delta t}\left[\frac{C_r}{B_o}+\frac{\partial(1/B_w)}{\partial P_w}\right]_{i,j}$$

$$C_{sww_{i,j}} = -\frac{\emptyset_{i,j}}{B_{w_{i,j}}\Delta t_{i,j}} - \left(\frac{dP_{cow}}{dS_w}\right)_{i,j} C_{pow_{i,j}}$$

Discrete form

Oil phase

$$\left(T_{x_o}\right)_{i+1/2,j}\left[(P_o)_{i+1,j} - (P_o)_{i,j}\right] + \left(T_{x_o}\right)_{i-1/2,j}\left[(P_o)_{i-1,j} - (P_o)_{i,j}\right] + \left(T_{y_o}\right)_{i,j+1/2}\left[(P_o)_{i,j+1} - (P_o)_{i,j}\right] + \left(T_{y_o}\right)_{i,j-1/2}\left[(P_o)_{i,j-1} - (P_o)_{i,j}\right] - q'_{o_{i,j}} = C_{poo_{i,j}}\left[(P_o)_{i,j}^{t+1} - (P_o)_{i,j}^{t}\right] + C_{swo_{i,j}}\left[(S_w)_{i,j}^{t+1} - (S_w)_{i,j}^{t}\right]$$

i= 1, N

Water phase

$$\left(T_{x_w}\right)_{i+1/2,j}\left[(P_o)_{i+1,j} - (P_o)_{i,j} - (Pc_{ow})_{i+1,j} + (Pc_{ow})_{i,j}\right] + \left(T_{x_w}\right)_{i-1/2,j}\left[(P_o)_{i-1,j} - (P_o)_{i,j} - (Pc_{ow})_{i-1,j} + (Pc_{ow})_{i,j}\right] + \left(T_{y_w}\right)_{i,j+1/2}\left[(P_o)_{i,j+1} - (P_o)_{i,j} - (Pc_{ow})_{i,j+1} + (Pc_{ow})_{i,j}\right] + \left(T_{y_w}\right)_{i,j-1/2}\left[(P_o)_{i,j-1} - (P_o)_{i,j} - (Pc_{ow})_{i,j-1} + (Pc_{ow})_{i,j}\right] - q'_{w_{i,j}} = C_{pow_{i,j}}\left[(P_o)_{i,j}^{t+1} - (P_o)_{i,j}^{t}\right] + C_{sww_{i,j}}\left[(S_w)_{i,j}^{t+1} - (S_w)_{i,j}^{t}\right]$$

Solution by IMPES method

Oil pressure, $(P_o)_{i,j}^t$ and water saturation, $(S_w)_{i,j}^t$ are the primary variables and unknowns to be solved for.

IMPES pressure solution

Pressure equation

Combine oil and water phase

$$\left[\left(T_{x_o}\right)_{i+1/2,j} + \alpha\left(T_{x_w}\right)_{i+1/2,j}\right]\left[(P_o)_{i+1,j} - (P_o)_{i,j}\right] + \left[\left(T_{x_o}\right)_{i-1/2,j} + \alpha\left(T_{x_w}\right)_{i-1/2,j}\right]\left[(P_o)_{i-1,j} - (P_o)_{i,j}\right] + \left[\left(T_{y_o}\right)_{i,j+1/2} + \alpha\left(T_{y_w}\right)_{i,j+1/2}\right]\left[(P_o)_{i,j+1} - (P_o)_{i,j}\right] + \left[\left(T_{y_o}\right)_{i,j-1/2} + \alpha\left(T_{y_w}\right)_{i,j-1/2}\right]\left[(P_o)_{i,j-1} - (P_o)_{i,j}\right] -$$

$$\alpha\left(T_{x_w}\right)_{i+1/2,j}[(Pc_{ow})_{i+1,j} + (Pc_{ow})_{i,j}] - \alpha\left(T_{x_w}\right)_{i-1/2,j}\left[(Pc_{ow})_{i-1,j} + (Pc_{ow})_{i,j}\right] -$$

$$\alpha\left(T_{y_w}\right)_{i,j+1/2}\left[(Pc_{ow})_{i,j+1} + (Pc_{ow})_{i,j}\right] - \alpha\left(T_{y_w}\right)_{i,j-1/2}\left[(Pc_{ow})_{i,j-1} + (Pc_{ow})_{i,j}\right] -$$

$$q'_{o_{i,j}} - \alpha q'_{w_{i,j}} = (C_{poo_{i,j}} + \alpha C_{pow_{i,j}})\left[(P_o)_{i,j}^{t+1} - (P_o)_{i,j}^{t}\right] \qquad \text{i=1,N}$$

Where

$$\alpha = -\frac{C_{sww_{i,j}}}{C_{swo_{i,j}}}$$

The pressure equation may now be rewritten as:

$$[\left(T_{x_o}\right)_{i-1/2,j} + \alpha\left(T_{x_w}\right)_{i-1/2,j}]\left[(P_o)_{i-1,j}\right] +$$

$$\left[\left(T_{x_o}\right)_{i+1/2,j} + \alpha\left(T_{x_w}\right)_{i+1/2,j}\right]\left[(P_o)_{i+1,j}\right] + \left[\left[-\left(T_{x_o}\right)_{i+1/2,j} + [\left(T_{x_o}\right)_{i-1/2,j} +\right.\right.$$

$$\left. C_{poo_{i,j}} + \left(T_{y_o}\right)_{i,j+1/2} + \left(T_{y_o}\right)_{i,j-1/2}\right] - \alpha[\left(T_{x_w}\right)_{i+1/2,j} + \left(T_{x_w}\right)_{i-1/2,j} +$$

$$\left.\left(T_{y_w}\right)_{i,j+1/2} + \left(T_{y_w}\right)_{i,j-1/2} + C_{pow_{i,j}}]\right][(P_o)_{i,j}^{t+1}] + [\left(T_{y_o}\right)_{i,j+1/2} +$$

$$\alpha\left(T_{y_w}\right)_{i,j+1/2}]\ [(P_o)_{i,j+1}] + [\left(T_{y_o}\right)_{i,j-1/2} + \alpha\left(T_{y_w}\right)_{i,j-1/2}]\left[(P_o)_{i,j-1}\right] =$$

$$\alpha\left(T_{x_w}\right)_{i+1/2,j}[(Pc_{ow})_{i+1,j} + (Pc_{ow})_{i,j}] - \alpha\left(T_{x_w}\right)_{i-1/2,j}\left[(Pc_{ow})_{i-1,j} + (Pc_{ow})_{i,j}\right] -$$

$$\alpha\left(T_{y_w}\right)_{i,j+1/2}\left[(Pc_{ow})_{i,j+1} + (Pc_{ow})_{i,j}\right] - \alpha\left(T_{y_w}\right)_{i,j-1/2}\left[(Pc_{ow})_{i,j-1} + (Pc_{ow})_{i,j}\right] +$$

$$q'_{o_{i,j}} + \alpha q'_{w_{i,j}} - [C_{poo_{i,j}} + \alpha C_{pow_{i,j}}][(P_o)_{i,j}^{t}]$$

In a simpler form

$$a\left[(P_o)_{i-1,j}\right] + b\left[(P_o)_{i+1,j}\right] + c\left[(P_o)_{i,j}\right] + d\left[(P_o)_{i,j+1}\right] + e\left[(P_o)_{i,j-1}\right] = f$$

$$a = [\left(T_{x_o}\right)_{i-1/2,j} + \alpha\left(T_{x_w}\right)_{i-1/2,j}]$$

$$b = \left[\left(T_{x_o}\right)_{i+1/2,j} + \alpha\left(T_{x_w}\right)_{i+1/2,j}\right]$$

$$c = \left[\left[-\left(T_{x_o}\right)_{i+1/2,j} + [\left(T_{x_o}\right)_{i-1/2,j} + C_{poo_{i,j}} + \left(T_{y_o}\right)_{i,j+1/2} + \left(T_{y_o}\right)_{i,j-1/2}\right] -\right.$$

$$\left.\alpha[\left(T_{x_w}\right)_{i+1/2,j} + \left(T_{x_w}\right)_{i-1/2,j} + \left(T_{y_w}\right)_{i,j+1/2} + \left(T_{y_w}\right)_{i,j-1/2} + C_{pow_{i,j}}]\right]$$

$$d = [\left(T_{y_o}\right)_{i,j+1/_2} + \alpha\left(T_{y_w}\right)_{i,j+1/_2}]$$

$$e = [\left(T_{y_o}\right)_{i,j-1/_2} + \alpha\left(T_{y_w}\right)_{i,j-1/_2}]$$

$$f = \alpha\left(T_{x_w}\right)_{i+1/_2,j}[(Pc_{ow})_{i+1,j} + (Pc_{ow})_{i,j}] - \alpha\left(T_{x_w}\right)_{i-1/_2,j}\left[(Pc_{ow})_{i-1,j} + (Pc_{ow})_{i,j}\right] - \alpha\left(T_{y_w}\right)_{i,j+1/_2}\left[(Pc_{ow})_{i,j+1} + (Pc_{ow})_{i,j}\right] - \alpha\left(T_{y_w}\right)_{i,j-1/_2}\left[(Pc_{ow})_{i,j-1} + (Pc_{ow})_{i,j}\right] + q'_{o_{i,j}} + \alpha q'_{w_{i,j}} - [C_{poo_{i,j}} + \alpha C_{pow_{i,j}}][(P_o)^t_{i,j}]$$

The equation obtained from above is the forward model from the derivation. Therefore, the equation will be generalized and adapted on numerical computational tool for automatic updating.

Nomenclature

P = Pressure	T= Transmissibility
λ = Mobility	q = Flow rate
k = Permeability	μ = Viscosity
β = Formation Volume Factor	φ = Porosity
C = Compressibility	t = Time
Pc_{OW} = Capillary pressure	